Individuals Round 1 States 2008

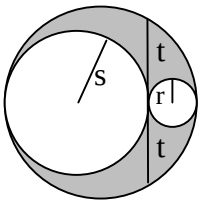
3 pts 1. Two opposite sides of a square are increased by 25%, while the other two sides are decreased by 40%, making a rectangle. Find the percent decrease from the area of the square to the area of the rectangle.

Ans. _____

4 pts 2. Find the following sum, leaving your answer in base eight:

$66_{\text{eight}} + 132_{\text{four}} + 1011_{\text{two}}$ **Ans. _____**

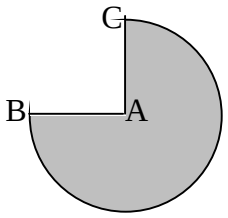
5 pts 3. The length of the chord tangent to the inner circles is 2t. The radius of larger inner circle is s, while the smaller one is r. Express the area of the region inside the largest circle and outside of the two inner circles as a function of t.



Ans. _____

Individuals Round 2 States 2008

3 pts 1. What is the area of the shaded region shown? Give exact answer.
Radius AC = 20 inches. $\overline{AB} \perp \overline{AC}$.



Ans. _____

4 pts 2. Find f(g(h(3)) given $f(x) = x^2 - 2x + 3$, $g(x) = 5 / x$, and $h(x) = \sqrt{x^2 - 5x + \frac{25}{4}}$

Ans. _____

5 pts 3. Using logarithms in base b, find the value(s) of b such that x + y = 72, given that $\log_b(xy) = 3$ and $\log_b(x/y) = 1$.

Ans. _____

Individuals Round 3 States 2008

3 pts 1. A computer game, Teacher Blaster, was on sale last week at 15% off the regular price. Then an additional 10% of the sale price was deducted to give a super sale price of $30.60. What was the regular price of the game in dollars?

Ans. _____

4 pts 2. Find the area bounded by the x-axis, the y-axis, and $(x-2)^2 + (y-2)^2 = 4$. **Ans. _____**

5 pts 3. After a committee meeting where ten people sat around a circular table, each person shook hands with everyone else except the people who sat on either side of them. How many handshakes took place?

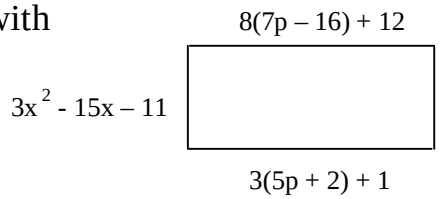
Ans. _____

Individuals Round 4 States 2008

3 pts 1. A dog chasing a rabbit, which has a head start of 150 ft, leaps 9 ft every time the rabbit hops 7ft. In how many leaps does the dog catch up to the rabbit, if the leaps and hops are made simultaneously, all in one direction?

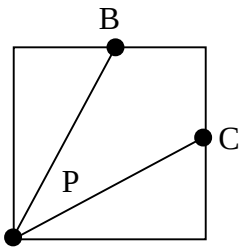
Ans. _____

4 pts 2. Find the area of the rectangle with the sides as indicated at right.



Ans. _____

5 pts 3. B and C are the midpoints of the sides of the square. Find sin P.



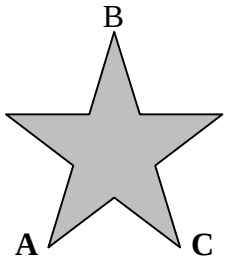
Ans. _____

Individuals Round 5 States 2008

3 pts 1. Suppose that $\frac{a+13b}{3a-b}=3$. Find the value of $\frac{a^3}{b^3}$.

Ans. _____

4 pts 2. In the regular pentagram, find the measure of angle ABC in degrees.



Ans. _____

5 pts 3. Let f(x) = cos x and let h be the smallest positive number so that the graph of y = f(x – h) will be symmetric about the y-axis. Find cos h.

Ans. _____

Individuals Round 6 States 2008

3 pts 1. If a & b = a^b + b, find (1 & 2) & 3

Ans. _____

4 pts 2. Solve for x: $\left(\frac{1}{3}\right)^{-1} \cdot 9^{x-1} = \left(\frac{1}{9}\right)^{2x-1}$

Ans. _____

5 pts 3. A sequence of numbers: a_1, a_2, a_3, a_4, … is formed according to the following rule: $a_1 = 19$, $a_2 = 77$, and for $n > 2$, $a_n = \frac{1-a_{n-1}}{a_{n-2}}$. What is the 2008^{th} term of the sequence?

Ans. _____

Team Round 1 States 2008

4 pts 1. Two different prime numbers are selected at random from the first 10 prime numbers. What is the probability the sum of the two primes is 24? Express your answer as a common fraction in lowest terms.

Ans. _____

4 pts 2. A triangle is bordered by three squares. If the areas of the squares are 225, 196, and 169, what is the area of the triangle?

Ans. _____

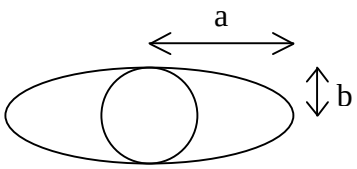
4 pts 3. Green Thumb wants to buy 100 plants for exactly $100.00, some plants at $0.95 each and the rest at $1.15 each. How many does she buy at $1.15.

Ans. _____

4 pts 4. All the house numbers on my side of the street are odd. My number has three digits and is a perfect square. Let P be the numbers that my house could have. What is the probability that if a number is chosen at random from P and its digits are reversed, it also makes a perfect square?

Ans. _____

6 pts 5. A circle is inscribed in an ellipse as shown, and the resulting three interior areas are all equal. What is the eccentricity of the ellipse given the area of an ellipse is π times the product of half the major axis and half the minor axis ($A = ab\pi$).



Ans. _____

6 pts 6. Find the area of a regular hexagon inscribed in a circle of radius 12 cm.

Ans. _____

6 pts 7. Let n be the smallest positive integer that is a multiple of 75 and has exactly 75 positive integral divisors, including 1 and itself. Find $n \div 75$.

Ans. _____

8 pts 8. Suppose that $\sec x + \tan x = {}^{22}/_{7}$ and that $\csc x + \cot x = {}^{m}/_{n}$ where ${}^{m}/_{n}$ is in lowest terms. Find $m + n$.

Ans. _____

8 pts 9. Given the sequence 6, 24, 60, 120, 210, … Find a formula for a_n in terms of n.

Ans. _____

Team Round 2 States 2008

4 pts 1. The last four digits of my telephone number add to six, but none of the digits is zero. What is the probability that you correctly guess these four digits of my number on the first try?

Ans. _____

4 pts 2. Solve for x: $\left|\frac{2}{3}x-\frac{4}{5}\right|<0.6$

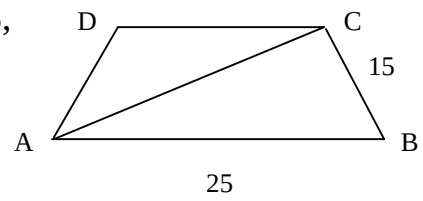
Ans. _____

4 pts 3. Find the units digit in the product of $\left(28^2\cdot 26^5\cdot 39^4\cdot 27^5\right)^{100}$. **Ans.** _____

4 pts 4. Two positive integral values of *k* exist for which the cubic polynomial $2x^3 - 9x^2 + 12x - k$ has a rational double root. What are these two values of *k*?

Ans. _____

6 pts 5. Given isosceles trapezoid ABCD with AB = 25, BC = 15, and $\overline{AC}\perp\overline{BC}$. Find the area of the trapezoid.



Ans. _____

6 pts 6. Find the ordered pair (x,y) such that $\frac{4}{x+y}+\frac{2}{x-y}=\frac{22}{15}$ and $\frac{6}{x+y}-\frac{5}{x-y}=\frac{-7}{15}$.

Ans. _____

6 pts 7. Evaluate $8+\cfrac{4}{8+\cfrac{4}{8+\cfrac{4}{8+\cdots}}}$

Ans. _____

8 pts 8. The sides of an equiangular hexagon measure 10, 6, 12, 14, x, and y units in the order given. Find the sum x + y.

Ans. _____

8 pts 9. If the sum of the series, 49 + 56 + 63 + … + x, is equal to 2,831,703, find x.

Ans. _____

Seat A Blue Relay Round 1 States 2008

My age is a multiple of 11, and I have gone through my teens. I am not over 100, and the digits of my age when multiplied make a perfect square and a perfect cube. How old am I?

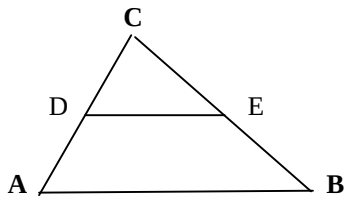
A = Your answer Pass back: A + 2

Seat B Blue Relay Round 1

The smallest angle of a triangle is 4^o smaller than the next larger angle. The third angle is 14^o less than three times the smallest angle. Find the measure of the largest angle.

B = Your answer X = The number you will receive. Pass back: $\frac{X+B}{2}$

Seat C Blue Relay Round 1



The area of triangle CDE = 28. D is the midpoint of $\overline{AC}$.
E is the midpoint of $\overline{BC}$. Find the area of quadrilateral ABED.

C = Your answer X = The number you will receive Pass back: 2(X –C)

Seat D Blue Relay Round 1

Solve for x: $\log_4(x-2) + \log_4(2x-5) = \log_4(7x-4)$.

D = Your answer X = The number you will receive Pass back: $\frac{X}{D-5}$.

Seat E Blue Relay Round 1

The 8th term of an arithmetic sequence is 27. The 15th term is 90. What is the 42nd term?

E = Your answer X = The number you will receive Pass in: $\frac{1}{3}E - 9X$

Seat A Green Relay Round 1 States 2008

I am not a teenager any longer. My age is a multiple of 13 and I am not 100 yet. Find my age if the product of the digits is a perfect square.

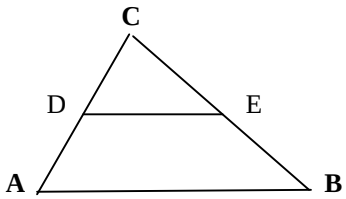
A = Your answer Pass back: A – 11

Seat B Green Relay Round 1

The smallest angle of a triangle is 4^o smaller than the next larger angle. The third angle is 14^o less than three times the smallest angle. Find the measure of the smallest angle.

B = Your answer X = The number you will receive Pass back: $\frac{X-B}{2}$

Seat C Green Relay Round 1



The area of triangle CDE = 58. D is the midpoint of $\overline{AC}$.
E is the midpoint of $\overline{BC}$. Find the area of quadrilateral ABED.

C = Your answer X = The number you will receive Pass back: C – 8X

Seat D Green Relay Round 1

Solve for x: $\log_6(x-2) + \log_6(2x-5) = \log_6(7x-4)$.

D = Your answer X = The number you will receive Pass back: $\frac{DX}{2}$

Seat E Green Relay Round 1

The 8^{th} term of an arithmetic sequence is 27. The 15^{th} is 90. What is the 23^{rd} term?

E = Your answer X = The number you will receive Pass in: 8X – E

Seat A Pink Relay Round 2 States 2008

Find the sum of the x-intercept, y-intercept and slope of the line $2x - 3y = 8$.

A = Your answer Pass back: 6A

Seat B Pink Relay Round 2

Ted can shovel a certain size roof off in 12 hours. Sam can do it in 8 hours. Ted shovels alone for 3 hours before Sam comes to help him finish. How many hours does Ted work?

B = Your answer X = the number you will receive Pass back: $\frac{5}{6}$BX

Seat C Pink Relay Round

The diagonals of a rhombus are 12 and 16 in length. What is the height of the rhombus?

C = Your answer X = The number you will receive Pass back: 10C - X

Seat D Pink Relay Round 2

Simplify: $\sqrt{\frac{3^{8x+12} \cdot 4^{x-5}}{9^{4x+4} \cdot 2^{2x-12}}}$

D = Your answer X = The number you will receive Pass back: $\frac{X - D}{2}$

Seat E Pink Relay Round 2

Nick has 3 dimes, 2 nickels, and 2 quarters in his pocket. He reaches in and takes out two coins. What is the probability that he has 35 cents in his hand?

E = Your answer X = The number you will receive Pass in: 14EX

Seat A Yellow Relay Round 2

Find the sum of the x-intercept, y-intercept, and slope of the line 5y + 3x = 18.

A = Your answer Pass back: 5A

Seat B Yellow Relay Round 2

Ted can shovel a certain size roof off in 12 hours. Sam can do it in 8 hours. Ted shovels alone for 3 hours before Sam comes to help him finish. How many hours does Sam work?

B = Your answer X = The number you receive Pass back: $\frac{1}{3}$BX

Seat C Yellow Relay Round 2

The diagonals of a rhombus are 6 and 8 units long. What is the height of the rhombus?

C = Your answer X = The number you receive Pass back: X – 10C

Seat D Yellow Relay Round 2

Simplify: $\sqrt{\frac{3^{6x+10} \cdot 8^{x-1}}{9^{3x+3} \cdot 2^{3x-7}}}$

D = Your answer X = The number you will receive Pass back: $\frac{D}{X}$

Seat E Yellow Relay Round 2

Nick has 3 dimes, 2 nickels, and 2 quarters. Nick reaches into his pocket for two coins. What is the probability that he has 15 cents in his hand?

E = Your answer X = The number you will receive Pass in: 21EX

Solutions – Individuals Round 1 States 2008

1. Area of original square is x^2. Dimensions of rectangle are 1.25 x and 0.6 x, so the area of the rectangle is 0.75 x^2. Percent of decrease is 25% **Ans. 25 or 25 %**

2. $66_8 = 54_{10}$, $132_4 = 30_{10}$, $1011_2 = 11_{10}$. 54 + 30 + 11 = 95. 95÷8 = 11 r 7, 11÷8 = 1 r 3, 1÷8 = 0 r 1, $95_{10} = 137_8$. **Ans. 137 or 137_{eight}**

3. Diameter of large circle: 2r + 2s. Radius of large circle is r + s. Shaded area = $(r + s)^2\pi - r^2\pi - s^2\pi = 2rs\pi$. Forming a right triangle with hypotenuse equal to the diameter of the large circle generates a right triangle with base (2r + 2s) and height t so $t^2 = 4rs$ and $rs = t^2/4$ and $2rs = 2t^2/4$ and $2rs\pi = t^2\pi/4$. **Ans.** $\frac{t^2 p}{2}$

Individuals Round 2 States 2008

1. The area of the circle is $20^2 p = 400p$. Area of region $300p$. **Ans. $300p$**

2. h(x) simplifies to $(x - {}^5/_2)$, so $h(3) = 3 - {}^5/_2 = ½$. $g(½) = 5 \div ½ = 10$. f(10) = 100 – 20 + 3 = 83. **Ans. 83**

3. log (xy) = 3 **è** (1) log x + log y = 3. log (x/y) = 1 **è** (2) log x – log y = 1. Adding (1) and (2): 2 log x = 4 **è** log x = 2, $x = b^2$. Substituting log x = 2 into (1): 2 + log y = 3 **è** log y = 1, y = b. Subbing this into x + y = 72: $b^2 + b = 72$ **è** $b^2 + b - 72 = 0$ **è** (b + 9)(b – 8) = 0 and b = -9 or 8. b > 0 so b = 8. **Ans. 8**

Individuals Round 3 States 2008

1. Let p = regular price, 0.9(0.85p) = 30.60 **à** 0.765 p = 30.60 **à** p = 40.00. **Ans. $40.00**

2. $(x - 2)^2 + (y - 2)^2 = 4$ is a circle with radius 2 centered at (2,2). The area sought is one corner of the square surrounding the circle. Area of square = 16. Area of circle = 4π. Area sought = ¼ $(16 - 4\pi) = 4 - \pi$ **Ans. $(4 - \pi)$ square units**

2. This problem is similar to the number of diagonals in a polygon of n sides: $\frac{n(n-3)}{2} = \frac{10(7)}{2} = 35$. **Ans. 35**

Individuals Round 4 States 2008

1. Let L = # of leaps. 9L = 7L + 150, 2L = 150, L = 75. **Ans. 75 or 75 leaps**

2. Equal widths produce: $3x^2 - 15x - 11 = x^2 - 5x + 17$ **è** $2x^2 - 10x - 28 = 0$ **è**

$x^2 - 5x - 14 = 0$ **è** $(x - 7)(x + 2) = 0$, so $x = 7$. Subbing in for width: $49 - 35 + 17 = 31$.

Equal lengths produce: $8(7p - 16) + 12 = 3(5p + 2) + 1$ **è** $56p - 128 + 12 = 15p + 6 + 1$ **è** $41p = 123$. Thus $p = 3$, so length $= 3(15 + 2) + 1 = 52$. Area $= 52(31) = 1612$.

Ans. 1612

3. Connecting B to C forms an isosceles right triangle with legs 1, if you let each side of the square be 2. Drawing a perpendicular from the vertex of P to $\overline{BC}$ divides $\overline{BC}$ into two equal lengths of $\sqrt{2}/2$ each. Now $\sin \frac{1}{2}P = \frac{\frac{\sqrt{2}}{2}}{\sqrt{5}} = \frac{\sqrt{10}}{10}$, so $\sqrt{\frac{1-\cos P}{2}} = \frac{\sqrt{10}}{10}$ Squaring: $\frac{1-\cos P}{2} = \frac{1}{10}$ **è** $1 - \cos P = \frac{1}{5}$. Thus $\cos P = \frac{4}{5}$ and $\sin P = \frac{3}{5}$. **Ans.** $\frac{3}{5}$

Individuals Round 5 States 2008

1. $\frac{a+13b}{3a-b} = 3$ **à** $a + 13b = 9a - 3b$ **à** $16b = 8a$, so $a = 2b$. $\frac{a}{b} = 2$, so $\frac{a^3}{b^3} = 8$. **Ans. 8**

2. Since it's a regular pentagram, the pentagon inside is regular with all angles equal to 108°. In each isosceles triangle, the base angles are supplementary to 108°, so they are 72°. The vertex angle $= 180 - 72 - 72 = 36$. **Ans. 36 or 36°**

3. The graph of $f(x) = \cos x$ starts out symmetric to the y-axis. Moving the graph π units causes it to be symmetrical to the y-axis again. $\cos \pi = -1$. **Ans. -1**

Individuals Round 6 States 2008

1. $(1 \& 2) = 1^2 + 2 = 3$. $(3 \& 3) = 3^3 + 3 = 30$. **Ans. 30**

2. $\left(\frac{1}{3}\right)^{-1} \times 9^{x-1} = \left(\frac{1}{9}\right)^{2x-1}$ **à** $3^1 \times \left(3^2\right)^{x-1} = \left(3^{-2}\right)^{2x-1}$ **à** $3 \times 3^{2x-2} = 3^{-4x+2}$, so $1 + 2x - 2 = -4x + 2$ and $6x = 3$ so $x = \frac{1}{2}$. **Ans. 1/2**

3. $a_1 = 19$, $a_2 = 77$, $a_3 = {}^{-76}/_{19} = -4$, $a_4 = {}^{5}/_{77}$, $a_5 = (1 - {}^{5}/_{77})/-4 = {}^{72}/_{77} * {}^{1}/_{-4} = {}^{-18}/_{77}$, $a_6 = (1 + {}^{18}/_{77})/{}^{5}/_{77} = {}^{95}/_{77} * {}^{77}/_{5} = 19$, $a_7 = (1\text{-}19)/{}^{-18}/_{77} = -18 * {}^{77}/_{-18} = 77$.

This sequence repeats every 5 terms, so the 2008^{th} term will correspond with the third term since $2008 \div 5$ has a remainder of 3. **Ans. -4**

Team Round I States 2008

1. 10 x 9 = 90 ways to pull two numbers. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 are the first 10 primes. (5, 19), (7,17), (11, 13), (19, 5), (17, 7), and (13,11) are six ways to succeed, so $^{6}/_{90} = {}^{1}/_{15}$. **Ans. 1/15**

2. Side lengths of the triangle are 13, 14, and 15. Hero's formula for the area of a triangle is: A = $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$, so A = $\sqrt{21\times8\times7\times6} = 84$.3 **Ans. 84 sq units**

3. 95(100-x) + 115x = 10000. 20x = 500. x = 25. **Ans. 25**

4. The three digit odd perfect squares are: 121, 169, 225, 289, 361, 441, 529, 625, 729, 841, 961 = P. Those which produce perfect squares when reversed are: 121, 169, 441, and 961. 4 out of 11. **Ans. 4/11**

5. The area of the circle is πb^2 and the area of the ellipse is πab. The eccentricity of an ellipse is $^{c}/_{a}$ and $a^2 = b^2 + c^2$. Circle's area = $^{1}/_{3}$ Ellipse's area so $\pi b^2 = {}^{1}/_{3}\ \pi ab$ and $b = {}^{1}/_{3}\ a$. This gives $a^2 = {}^{1}/_{9}\ a^2 + c^2$ so $^{8}/_{9}\ a^2 = c^2$ **è** $8/9 = {}^{c^2}/_{a^2}$ **è** $\frac{2\sqrt{2}}{3} = {}^{c}/_{a}$ $\frac{2\sqrt{2}}{3}$ **Ans.** $\frac{2\sqrt{2}}{3}$

6. The hexagon is made up of six equilateral triangles with a side length of 12 cm. The area each triangle = $\frac{s^2\sqrt{3}}{4}$, 6 * $\frac{s^2\sqrt{3}}{4}$ = $6\times\frac{12^2\sqrt{3}}{4} = 6\times12\times3\times\sqrt{3}$ =$216\sqrt{3}$. **Ans.** $216\sqrt{3}$

7. $75 = 3 \times 5^2$. To have 75 positive factors, the exponents on the prime factors must be 2, 4, and 4 since adding one to each exponent, then multiplying the result: 3(5)(5) = 75. For this number to be the smallest multiple of 75, using 2, 3, and 5 as the smallest prime factors and raising 2 to the 4th power, 3 to the 4th power, and 5 to the 2nd power generates the smallest multiple of 75 with 75 positive factors: $2^4 \times 3^4 \times 5^2 = 32400$. 32400 ÷ 75 = 432. **Ans. 432**

8. sec x + tan x = $^{22}/_{7}$ **è** $\frac{1}{\cos x} + \frac{\sin x}{\cos x} = \frac{22}{7}$ **è** $\frac{1+\sin x}{\cos x} = \frac{22}{7}$ **è** $\frac{1+\sin x}{\sqrt{1-\sin x}} = \frac{22}{7}$. Thus: $7(1 + \sin x) = 22\sqrt{1-\sin x}$. Squaring: $49 + 98 \sin x + 49 \sin^2 x = 484 - 484 \sin^2 x$. $533 \sin^2 x - 968 \sin x + 435 = 0$. Factoring: $(533 \sin x - 435)(\sin x - 1) = 0$. Thus $\sin x = 1$, which it does not, or $\sin x = \frac{435}{533}$. Thus $\csc x = \frac{533}{435}$ and $\cot x = \frac{308}{435}$. $\csc x + \cot x = {}^{533}/_{435} + {}^{308}/_{435} = {}^{841}/_{435} = {}^{29}/_{15} = {}^{m}/_{n}$. So $m + n = 29 + 15 = 44$. **Ans. 44**

9. 6 = 1 x 2 x 3, 24 = 2 x 3 x 4, 60 = 3 x 4 x 5, 120 = 4 x 5 x 6, 210 = 5 x 6 x 7, so a_n = n(n+1)(n+2). **Ans.** n(n+1)(n+2)

1. There are four possible numbers using 3, 1, 1, 1 and six possible numbers using 2, 2, 1, 1. **Ans. 1/10**

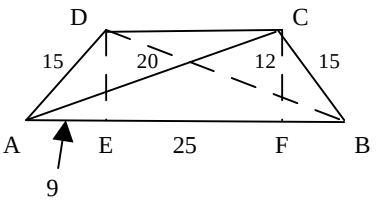
2. $\left|\frac{2}{3}x-\frac{4}{5}\right|<0.6$ **è** $-0.6<\frac{2}{3}x-\frac{4}{5}<0.6$ **è** $-9 < 10x - 12 < 9$ **è** $3 < 10x < 21$.
Thus $^{3}/_{10} < x < {}^{21}/_{10}$. **Ans.** $^{3}/_{10} < x < {}^{21}/_{10}$

3. The unit's digit of the inside $\left(28^2 \cdot 26^5 \cdot 39^4 \cdot 27^5\right)$ is $(6 \cdot 6 \cdot 1 \cdot 7) = 2$. The sequence when the unit's digit is 2 is 2, 4, 8, 6, 2, 4 etc. Then the units for $\left(28^2 \cdot 26^5 \cdot 39^4 \cdot 27^5\right)^{100} = 6$. **Ans. 6**

h(x) simplifies to $(x - {}^{5}/_{2})$, so $h(3) = 3 - {}^{5}/_{2} = ½$. $g(½) = 5 \div ½ = 10$. $f(10) = 100 - 20 + 3 = 83$.

4. For $y = 2x^3 - 9x^2 + 12x - k$, the sum of the three roots is $^{9}/_{2}$. Given the signs of the terms, all of the roots must be positive. One possibility for the 3 roots is: $1 + 1 + {}^{5}/_{2}$ with a product of $^{5}/_{2}$. The only other possibility is: $2 + 2 + ½$ with a product of 2. The product of the roots is $^{k}/_{2}$. So for the first possibility, $^{5}/_{2} = {}^{k}/_{2}$ and $k = 5$. For the second possibility, $^{k}/_{2} = 2$ and $k = 4$. **Ans. 4, 5 or k = 4 or 5**

5. Connect B to D also. By the Pythag. Thm AC and BD = 20. Drop perpendiculas from C and D to AB to meet at F and E respectively. Then the area of ΔABC is ½ 15(20) = ½ 25(CF). Thus CF = 12 and so does DE. Again By the Pyth. Thm. AE = BF = 9. Thus EF = 7, and so does DC. The area of the trapezoid = ½ (12)(25 + 7) = 6(32) = 192. **Ans. 192**

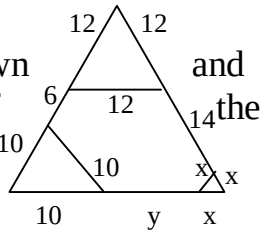


6. Let (1) $\frac{1}{x+y}=a$ and (2) $\frac{1}{x-y}=b$. Then $\frac{4}{x+y}+\frac{2}{x-y}=\frac{22}{15}$ will be (3) $4a + 2b = \frac{22}{15}$ and $\frac{6}{x+y}-\frac{5}{x-y}=\frac{-7}{15}$ will be (4) $6a - 5b = -\frac{7}{15}$. Multiplying (3) by 5/2 and adding to (4): $16a = \frac{48}{15}$ **è** $a = \frac{1}{5}$. In (4): $\frac{6}{5}-5b=-\frac{7}{15}$ **è** $-5b = -\frac{25}{15}$, so $b = \frac{1}{3}$. Thus in (1) and (2): $x + y = 5$ and $x - y = 3$. $2x = 8$, so $x = 4$. $2y = 2$, so $y = 1$. **Ans. (4, 1)**

7. Let $x = 8+\cfrac{4}{8+\cfrac{4}{8+\cfrac{4}{8+\cdots}}}$ then $x = 8 + \frac{4}{x}$ **è** $x^2 = 8x + 4$ **è** $x^2 - 8x - 4 = 0$. Thus:

$x = \frac{8 \pm \sqrt{64-4(-4)}}{2} = \frac{8 \pm \sqrt{80}}{2} = \frac{8 \pm 4\sqrt{5}}{2} = 4 \pm 2\sqrt{5}$. Only answer $4 \pm 2\sqrt{5}$. **Ans.** $4 + 2\sqrt{5}$

8. Extend the sides of the hexagon to form an equilateral triangle as shown and note that the smaller triangles formed are also equilateral. (Each angle of the hexagon is 120° - its supplement would have measure 60°.) Then by the equality of the three sides of the large outer triangle, x + y + 10 = 12 + 6 + 10, so x + y = 18. **Ans. 18**



9. The last term x = 49 + (n-1)7 **è** $\frac{x}{7}$ = 7 + n-1. So n = $\frac{x}{7}$ - 6.

49 + 56 + 63 + … + x = (49 + x) (x÷7 – 6) ÷ 2 = 2,831,703, so 7x + x² ÷ 7 – 294 – 6x = 5663406 **è** Mult. 7: 0 = x² + 7x – 39645900 **è** (x + 6300)(x – 6293) = 0, so x = 6293. **Ans. 6293**

Seat A Blue Relay Round 1 States 2008

Of the numbers 22, 33, 44, 55, 66, 77, 88. 99. 88 **è** 8(8) = 64. **Ans. A = 88**

Pass back: A + 2 = 88 + 2 **= 90**

Seat B Blue Relay

x – 4 + x + 3(x-4) – 14 = 180 **è** 5x – 30 = 180 **è** 5x = 210. x = 42. 3(42-4)-14 = 100

B = 100. **Pass back:** $\frac{X+B}{2}=\frac{90+100}{2}$ **= 95** **Ans. B = 100**

Seat C Blue Relay

The ratio of the areas of similar triangles CDE and ABC is the square of the ratio of the sides $(1/2)^2$ = ¼. So the area of quadrilateral ABED is 3 times the area of triangle CDE = 84. C = 84. **Pass back:** 2(X – C) = 2(95 – 84) **= 22** **Ans. C = 84**

Seat D Blue Relay

(x – 2)(2x -5) = 7x – 4 **è** $2x^2$ - 9x + 10 = 7x – 4 **è** $2x^2$ - 16x + 14 = 0 **è** x^2 - 8x + 7 = 0

(x – 7)(x – 1) = 0. x = 7 or 1. But x cannot = 1. **Ans. D = 7**

Pass back: $\frac{X}{D-5}=\frac{22}{7-5}$ = **11**

Seat E Blue Relay

(1) a + 7d = 27 and (2) a + 14d = 90. (2) – (1): 7d = 63. So d = 9. Subbing into (1): a + 63 = 27, a = -36. The 42nd term: -36 + 41(9) = 369 – 36 = 333. **Ans. E = 333**

Pass back: $\frac{1}{3}E-9X=\frac{1}{3}(333)-9(11)$ = 111 – 99 = **12**

Seat A Green Relay States 2008

91 **è** (9)(1) = 9 a perfect square. **Ans. A = 91**

Pass back: A – 11 **è** 91 – 11 = **80**

Seat B Green Relay

From Blue Seat A smallest angle = 38. **Ans. B = 38**

Pass back: (X – B)/2 = (80 – 38)/2 = **21**

Seat C Green Relay

From Blue Relay Seat C: 3(58) = 174 **Ans. C = 174**

Pass back: C – 8X = 174 – 8(21) = 174 – 168 = **6**

Seat D Green Relay

Same as Blue Seat D different base has no effect: 7 **Ans. D = 7**

Pass back: DX/2 = (7)(6)/2 = **21**

Seat E Green Relay

From Blue E: 23^{rd} term = -36 + 22(9) = -36 + 198 = 162. **Ans. E = 162**

Pass back: 8X – E = 8(21) – 162 = 168 – 162 = **6**

Seat A Pink Relay States 2008

For 2x – 3y = 8, x-*i* = 4, y-*i* = -2⅔, slope = ⅔. The sum is 2. **Ans. A = 2**

Pass back: 6A = 6(2) = **12**

Seat B Pink Relay

$\frac{1}{12}(3 + t) + \frac{1}{8}t = 1$ **è** 2(3 + t) + 3t = 24 **è** 5t = 18 **è** t = 3⅗, 3 + t = 6⅗. **Ans. B = 6⅗**

Pass back: $\frac{5}{6}$BX = $\frac{5}{6}$(33⅗)(12) = **66**

Seat C Pink Relay

Using the right triangle made by ½ the diagonals makes a 6-8-10 Δ. Making 10 the side of the rhombus. The area of the rhombus is ½ $d_1 d_2$ = bh, thus ½ (12)(16) = 10h **è** h = 9.6

Pass Back: 10C – X = 10(9.6) – 66 = **30** **Ans. C = 9.6**

Seat D Pink Relay

$\sqrt{\frac{3^{8x+12} \cdot 4^{x-5}}{9^{4x+4} \cdot 2^{2x-12}}} = \sqrt{\frac{3^{2(4x+6)} \cdot 2^{2(x-5)}}{3^{2(4x+4)} \cdot 2^{2x-12}}} = \sqrt{\frac{3^{12} \cdot 2^{-10}}{3^{8} \cdot 2^{-12}}} = \sqrt{3^4 \cdot 2^2} = 18.$ **Ans. D = 18**

Pass back: (X – D)/2 = (30 – 18)/2 = **6**

Seat E Pink Relay

Total possibilities: $_7C_2$ = 21. Ways to get 35 cents: 3(2). Prob. = 6⁄21 = ⅔. **Ans. E =** ⅔

Pass back: 14EX = 14(⅔)(6) = **24**

Seat A Yellow Relay States 2008

For 5y + 3x = 18, x-*i* = 6, y-*i* = 3⅗, slope = -⅗. The sum = 9. **Ans. A = 9**

Pass back: 5A = 5(9) = **45**

Seat B Yellow Relay

In Pink A Sam works 3⅗ hours. **Ans. B** = 3⅗

Pass back: $\frac{1}{3}$BX = $\frac{1}{3}$(18⅗)(45) = **54**

Seat C Yellow Relay

Same as Blue A: ½ 6(8) = 5h **è** 5h = 24, h = 4.8 **Ans. C = 4.8**

Pass back: X – 10C = 54 – 10(4.8) = **6**

Seat D Yellow Relay

$$\sqrt{\frac{3^{6x+10}\cdot 8^{x-1}}{9^{3x+3}\cdot 2^{3x-7}}} = \sqrt{\frac{3^{2(3x+5)}\cdot 2^{3x-3}}{3^{2(3x+3)}\cdot 2^{3x-7}}} = \sqrt{\frac{3^{10}\cdot 2^{-3}}{3^{6}\cdot 2^{-7}}} = \sqrt{3^4\cdot 2^4} = 36$$ **Ans. D = 36**

Pass back: $\frac{D}{X} = \frac{36}{6}$ **= 6**

Seat E Yellow Relay

To get 15 cents: 3(2) = 6. Total possible outcomes 21 (Pink E) **Ans. E = 2/7**

Pass back: 21EX = 21(2/7)6 = **36**

Answer Sheet States 2008

Individuals – Round 1

1. **25 or 25 % or -25%**
2. **137 or 137_8**
3. $\frac{t^2p}{2}$ or $.5t^2p$

Individuals – Round 2

1. **$300p$ or $300p$ sq in or $300p$** sq units
2. **83**
3. **8**

Individuals – Round 3

1. **$40.00**
2. **$4 - p$ or $(4 - \pi)$ square units or $4 - p$ square units**
3. **35**

Individuals – Round 4

1. **75 or 75 leaps**
2. **1612**
3. $\frac{3}{5}$ or .6

Individuals – Round 5

1. **8**
2. **36 or 36^o**
3. **-1**

Individuals – Round 6

1. **30**
2. **½ or .5**
3. **-4 (no solution was accepted at the meet because of a type in the original problem)**

Team – Round 1

1. **1/15 or .0667**
2. **84 or 84 sq units**
3. **25**
4. **4/11 or .3636**
5. $\frac{2\sqrt{2}}{3}$ **or .9428**
6. $216\sqrt{3}$ **or 374.1230**
7. **432**
8. **44**
9. **$n(n + 1)(n + 2)$ or $n^3 + 3n^2 + 2n$**

Team – Round 2

1. **1/10 or .1 or 10%**
2. $^3/_{10} < x < {}^{21}/_{10}$ or $.3 < x < 2.1$
3. **6**
4. **4, 5**
5. **192**
6. **(4, 1)**
7. $4 + 2\sqrt{5}$ **or 8.4721**
8. **18**
9. **6293**

Blue Relay		Pass	Green Relay		Pass	Pink Relay		Pass	Yellow Relay		Pass
A	88	90	A	91	80	A	2	12	A	9	45
B	100	95	B	38	21	B	$6\frac{3}{5}$	66	B	$3\frac{3}{5}$	54
C	84	22	C	174	6	C	9.6	30	C	4.8	6
D	7	11	D	7	21	D	18	6	D	36	6
E	333	12	E	162	6	E	$\frac{2}{7}$	24	E	$\frac{2}{7}$	36