

**Individuals – Round 1 States 2019**

**3 pts 1.** From what polynomial must  $5x^3 - 2x^2 + 3x$  be subtracted in order to obtain  $x^3 - 2$ ?

**Ans.** \_\_\_\_\_

**4 pts 2.** Simplify:  $\frac{80 - 3[2^3 - (-4 - (-1)) + 3^2]}{-8 \div 4 \cdot 2}$ .

**Ans.** \_\_\_\_\_

**5 pts 3.** Find the sum of the integral solutions to  $|x - 45| < 23$ .

**Ans.** \_\_\_\_\_

**Individuals – Round 2 States 2019**

**3 pts 1.** A line crosses the y-axis at  $(a + 2010, a + 2011)$ . What is the value of  $a$ ?

**Ans.** \_\_\_\_\_

**4 pts 2.** Points L and U lie on the segment KN.  $\frac{LK}{LN} = \frac{4}{1}$  and  $\frac{UN}{UK} = \frac{5}{3}$ . If the fully reduced ratio of the length of LU to KN is  $p:q$ , where  $p$  and  $q$  are relatively prime, what is the numerical value of  $p + q$ ?

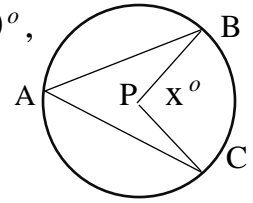
**Ans.** \_\_\_\_\_

**5 pts 3.** What is the maximum possible value of  $\frac{|7 + 24i|}{|a + 4i|}$ , where  $a$  is a real number?

**Ans.** \_\_\_\_\_

**Individuals – Round 3 States 2019**

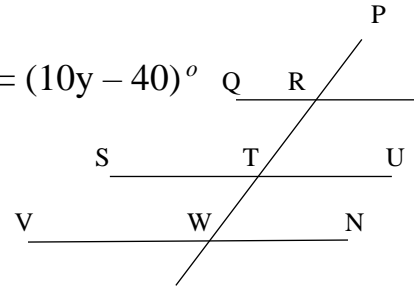
**3 pts 1.** Point P is the center of the circle,  $m\angle ABP = 30^\circ$ ,  $m\angle ACP = 20^\circ$ , and  $m\angle BPC = x^\circ$ . Find the value of x.



**Ans.** \_\_\_\_\_

**4 pts 2.** Given  $\overline{QR} \parallel \overline{ST} \parallel \overline{VW}$ ,  $m\angle PRQ = (8x + 14)^\circ$ ,  $m\angle UTW = (10y - 40)^\circ$

And  $m\angle TWN = (3x + y + 19)^\circ$ . Find  $m\angle QRT - m\angle RTS$ .



**Ans.** \_\_\_\_\_

**5 pts 3.** Let N be the smallest positive integer which ends with the number 17, which is divisible by 17 and whose digits have a sum of 17. Find the product of the digits of N.

**Ans.** \_\_\_\_\_

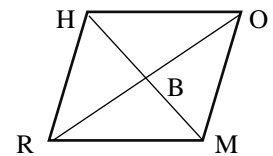
**Individuals – Round 4 States 2019**

**3 pts 1.** A math-magic trick using natural numbers is performed as follows: pick any number, add 5, multiply this answer by 6, subtract 15 from this new answer, divide this answer by 3, subtract the original number from this answer, then perform one final operation on this answer. What should the final operation be so that the result of this trick is always the original number? Write the operation and the number.

**Ans.** \_\_\_\_\_

**4 pts 2.** RHOM is a rhombus with  $RH = x + 2$ ,  $BH = x$  and  $BO = x + 1$ .

Find the area of Quadrilateral RHOM.



**Ans.** \_\_\_\_\_

**5 pts 3.** If  $\frac{x^{a^2}}{x^{b^2}} = x^{28}$ ,  $x > 1$  and  $a + b = 4$ , what is the value of  $a - b$ ?

**Ans.** \_\_\_\_\_

### Individuals – Round 5 States 2019

**3 pts 1.** Let a be defined as  $a * b = ab - b^a$ . Find the value of  $4 * (2 * 3)$ .

**Ans.** \_\_\_\_\_

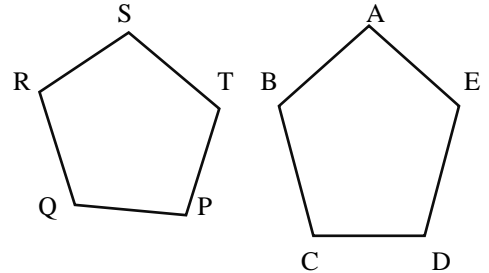
**4 pts 2.** For all  $x$ ,  $x \neq 0$  and real numbers  $a$ ,  $b$ , and  $c$ ,  $(ax + 3)(bx + 2) = 15x^2 + cx + 6$  and  $a + b = 16$ . Find 2 possible values of  $c$ .

**Ans.** \_\_\_\_\_

**5 pts 3.** Pentagon ABCDE is similar to pentagon PTSRQ.

$AB = 20$ ,  $BC = 24$ ,  $CD = 18$ ,  $DE = 25$ ,  $AE = 22$ , and  $RS = 20$ .

Find the perimeter of pentagon PTSRQ.



**Ans** \_\_\_\_\_

### Individuals – Round 6 States 2019

**3 pts 1.** Two sides of an isosceles triangle are 4 and 12. What is the perimeter of the triangle?

**Ans.** \_\_\_\_\_

**4 pts 2.** The graph of  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$  has asymptotes with equations  $2x + 3y = -4$  and  $2x - 3y = 8$ . Find the value of  $h - k$ .

**Ans.** \_\_\_\_\_

**5 pts 3.** Find all values of  $x$  for which  $\log_{13}(x^3 - 1) + \log_{\frac{1}{13}}(x - 1) = 1$ .

**Ans.** \_\_\_\_\_

Team – Round 1 States 2019

**4 pts 1.**  $\frac{(xy-z)(xy+z)+z^2}{yz^3}$  can be written in the form  $x^A y^B z^C$ .

Find the value of  $A + B - C$ .

**(1) Ans.** \_\_\_\_\_ **4 pts**

**4 pts 2.** Find the sum of the negative integers which are in the solution set for the inequality  $8 - \frac{n}{3} > 1 - 2(3 + n)$ .

**(2) Ans.** \_\_\_\_\_ **4 pts**

**6 pts 3.** In a game the score is obtained by subtracting the number of incorrect answers from twice the number of correct answers. If Mark answered 40 questions and obtained a score of 50, how many questions did Mark answer correctly.

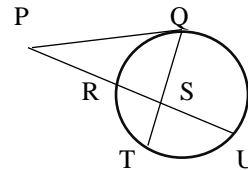
**(3) Ans.** \_\_\_\_\_ **6 pts**

**6 pts 4.** The Islesford School District formed a committee of 15% parents, 45% teachers, 25% school and district administrators and the remaining 6 are students. How many more teachers were on the committee than school and district administrators?

**(4) Ans.** \_\_\_\_\_ **6 pts**

**6 pts 5.**  $\overline{PQ}$  is tangent to the circle at Q and 6 units long.

$RS = 2$ ,  $PR = 3$ ,  $ST = 4$ ,  $QS = b$  and  $SU = a$ . Find the value of  $4a - 2b$ .



**(5) Ans.** \_\_\_\_\_ **6 pts**

**8 pts 6.** How many distinct ordered pairs of positive integers  $(x, y)$  satisfy  $x^y = 2^{20}$ .

**(6) Ans.** \_\_\_\_\_ **8 pts**

**8 pts 7.** The sum of the first  $n$  positive integers is 150 less than the sum of the first  $3n$  positive integers. What is the sum of the first  $5n$  positive integers?

**(7) Ans.** \_\_\_\_\_ **8 pts**

**8 pts 8.** Find the sum of all values of  $x$ , where  $0^\circ \leq x < 360^\circ$ , for which

$$\sec^2 x + 2 \cos x \sin x = 2 + \tan^2 x.$$

**(8) Ans.** \_\_\_\_\_ **8 pts**

Team – Round 2 States 2019

**4 pts 1.** Express  $4(a^2 - 2b + 5) - 3(4a^2 + b - 7)$  in terms of  $b$ , if  $a = -2b$ .

(1) Ans. \_\_\_\_\_ 4 pts

**4 pts 2.** In a large room, the ratio of 8<sup>th</sup> graders to 9<sup>th</sup> graders is 4 to 3. The ratio of 10<sup>th</sup> graders to 11<sup>th</sup> graders is 3 to 2. The ratio of 11<sup>th</sup> graders to 9<sup>th</sup> graders is 1 to 6. What is the ratio of 8<sup>th</sup> graders to 10<sup>th</sup> graders?

(2) Ans. \_\_\_\_\_ 4 pts

**6 pts 3.** Solve for  $x$ , if  $\frac{x^2 - 3x - 10}{x^2 + 3x - 28} \cdot \frac{x^2 + 8x + 7}{x^2 - x + 20} = 1$ .

(3) Ans. \_\_\_\_\_ 6 pts

**6 pts 4.** There are 6 males and 4 females on a committee. 3 are chosen at random. What is the probability that 2 males and 1 female are ones chosen?

(4) Ans. \_\_\_\_\_ 6 pts

**6 pts 5.** A grain silo is built in the shape of a right-circular cylinder with two right-circular cones attached, one on the top and one on the bottom. The height of the cylinder is 10 ft. and the total height of the silo is 20 ft. The radius of the cylinder is 5 ft. Find the volume of the silo in cubic feet.

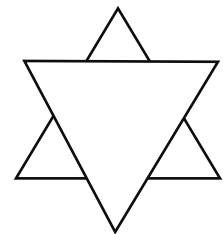
(5) Ans. \_\_\_\_\_ 6 pts

**8 pts 6.** Find the number of ordered pairs  $(a, b)$  of positive integers

for which  $2a + b \leq 9$ .

(6) Ans. \_\_\_\_\_ 8 pts

**8 pts 7.** Two congruent equilateral triangles, whose perimeters each equal 36, overlap to form a 6-pointed star. All six petals of the star are congruent. What is the area of the concave dodecagon?



(7) Ans. \_\_\_\_\_ 8 pts

**8 pts 8.** Find all real values of  $x$ , such that  $\begin{vmatrix} x & 5 & x+3 \\ 3 & x-2 & 6 \\ -5 & -x & x-3 \end{vmatrix} = -75$ .

(8) \_\_\_\_\_ 8 pts

**Seat A Blue Relay States 2019**

If  $a = b^2 - c$ ,  $b = \frac{c}{2} + 1$  and  $c = -3^3 + 23$ , find the numerical value of  $a + b$ .

Pass back:  $2A^2$        $A = \text{Your answer}$

**Seat B Blue Relay**

I have \$17 consisting of silver dollars and quarters. What is the value of my quarters in dollars, if I have 26 coins in all?

Pass back:  $B + X + 2$        $B = \text{Your answer}$        $X = \text{The number you will receive}$

**Seat C Blue Relay**

An angle's complement is  $54^\circ$  less than eight times the angle. What is the value of the angle's complement in degrees?

Pass back:  $\sqrt{\frac{32C}{X}}$        $C = \text{Your answer}$        $X = \text{The number you will receive}$

**Seat D Blue Relay**

If  $\log_3 6 = x$ , what is the value of  $9^{2x-1}$ ?

Pass back:  $D - 2X$        $D = \text{Your answer}$        $X = \text{The number you will receive}$

**Seat E Blue Relay**

For two acute angles  $a$  and  $b$ ,  $\sin a = \cos b$ . If  $a = 4k - 22$  and  $b = 6k - 13$ , what is the value of  $10k$ ?

Pass in:  $(X - E)^2$        $E = \text{Your answer}$        $X = \text{The number you will receive}$

**Seat A Green Relay States 2019**

If  $a = b^2 - c$ ,  $b = \frac{c}{2} + 1$  and  $c = -5^2 + 19$ , find the value of  $a + b$ .

Pass back:  $13A + 1$

A = Your answer

**Seat B Green Relay**

In my coin dish I have 90 coins consisting of only dimes and quarters. If there is \$18 in my coin dish, what is the value in dollars of all my quarters?

Pass back:  $\frac{X}{B}$

B = Your answer

X = The number you will receive

**Seat C Green Relay**

An angle's supplement is  $36^\circ$  more than seven times the angle. What is the measure of the angle?

Pass back:  $1000(C + X)$

C = Your answer

X = The number you will receive

**Seat D Green Relay**

If  $\log_3 10 = x$ , what is the value of  $27^{x+1}$ ?

Pass back:  $\frac{D - X}{400}$

D = Your answer

X = The number you will receive

**Seat E Green Relay**

For two acute angles  $a$  and  $b$ ,  $\sin a = \cos b$ . If  $a = 7k - 21$  and  $b = 4k + 12$ , what is the value of  $k$ ?

Pass in:  $X^{(E-X)}$

E = Your Answer

X = The number you will receive

### Seat A Pink Relay States 2019

Gena reduced  $\sqrt{605}$  correctly to  $a\sqrt{b}$ . However, she wrote her answer as  $b\sqrt{a}$ . Find  $c$ , if  $b\sqrt{a} = \sqrt{c}$ .

Pass back: A – 19

A = Your answer

### Seat B Pink Relay

The sum of the numerator and denominator of a certain fraction is 27. If 4 is subtracted from the numerator and 5 is added to the denominator, the fraction will equal  $\frac{1}{3}$ , when reduced. Find the original fraction.

Pass back: BX – 66

B = Your answer

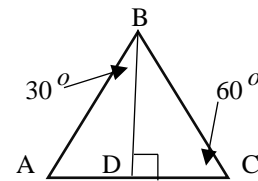
X = The number you will receive

### Seat C Pink Relay

In triangle ABC, find  $(BD)^2$ .  $\overline{BD} \perp \overline{AC}$  and  $BC = 12$ .

Pass back:  $(X - C)^5$

C = Your answer



X = The number you will receive

### Seat D Pink Relay

The minimum value of the function  $f(x) = 2x^2 - 8x + k$  is 24. Find the value of  $k$ .

Pass back:  $2\sqrt{DX}$

D = Your answer

X = The number you will receive

### Seat E Pink Relay

$\begin{bmatrix} \frac{x}{2} & 1 & 1 \\ 2 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix} = M$  and  $|M| = -16$ . Solve for  $x$ .

Pass in:  $7\sqrt[4]{X}$

E = Your answer

X = The number you will receive



**Seat A Yellow Relay States 2019**

Gene reduced  $\sqrt{245}$  correctly as  $a\sqrt{b}$ . However, he wrote his answer incorrectly as  $b\sqrt{a}$ .

If  $b\sqrt{a} = \sqrt{c}$ , find  $c$ .

Pass back: A – 15      A = Your answer

**Seat B Yellow Relay**

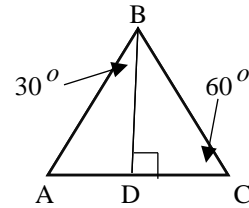
The sum of the numerator and denominator of a certain fraction is 47. If 2 is subtracted from the denominator and 5 is added to the numerator, the fraction will equal  $\frac{2}{3}$ , when reduced. Find the original fraction.

Pass back: BX      B = Your answer      X = The number you will receive

**Seat C Yellow Relay**

In  $\triangle ABC$ ,  $\overline{BD} \perp \overline{AC}$ ,  $BD = 6\sqrt{3}$ . Find the perimeter of  $\triangle ABC$ .

Pass back:  $\left(\frac{X}{\sqrt{C}+9}\right)^2$       C = Your answer



X = The number you will receive

**Seat D Yellow Relay**

The minimum value of the function  $f(x) = 2x^2 + 8x + k$  is -8. What is the value of  $k$ ?

Pass back: D + X + 2      D = Your answer      X = The number you will receive

**Seat E Yellow Relay**

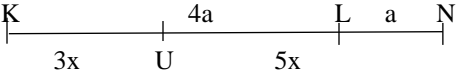
$\begin{bmatrix} \frac{x}{2} & 1 & 3 \\ -3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -2 & 2 \end{bmatrix} = M$  and  $|M| = 23$ . Solve for  $x$ .

Pass in:  $\left(\frac{X}{E-1}\right)^4$       E = Your answer      X = The number you will receive

## Solutions – Individuals Round 1

1.  $(5x^3 - 2x^2 + 3x) + (x^3 - 2) = 6x^3 - 2x^2 + 3x - 2$ . **Ans.  $6x^3 - 2x^2 + 3x - 2$**
2.  $\frac{80 - 3[2^3 - (-4 - (-1)) + 3^2]}{-8 \div 4 \cdot 2} = \frac{80 - 3[8 + 3 + 9]}{-2 \cdot 2} = \frac{20}{-4} = -5$ . **Ans. - 5**
3. (1)  $x - 45 < 23$  and (2)  $-x + 45 < 23$ . In (1)  $x < 68$ . In (2)  $x > 22$ . Thus the sum of the whole numbers from 23 to 67:  $45\left(\frac{23+67}{2}\right) = 45(45) = 2025$ . **Ans. 2025**

## Individuals Round 2

1. A line crosses the y-axis at  $x = 0$ . Thus  $a + 2010 = 0$ , so  $a = -2010$ . **Ans. - 2010**
2. The 4:1 ratio and the 5:3 ratio have totals of 5 and 8. So using the LCM of 40.  $4a + a = 40$ , and  $5x + 3x = 40$ .  $a = 8$ , and  $b = 5$ . So  $LU:KN = (40 - 8 - 15):40 = 17:40$ .  $p + q = 57$ . **Ans. 57**
- 
3.  $\frac{|7 + 24i|}{|a + 4i|} = \frac{\sqrt{7^2 + 24^2}}{\sqrt{a^2 + 4^2}} = \frac{25}{\sqrt{a^2 + 16}}$ . The denominator is smallest when  $a = 0$ . **Ans. 25/4**

## Individuals Round 3

1. Connect A to P.  $\square PAB = 30$ ,  $\square CAP = 20$ . So  $\square BAC = 50$ , thus  $\square BC = 100 = x$ . **Ans. 100**
2.  $8x + 14 = 10y - 40 \rightarrow (1) 4x - 5y = -27$ .  $10y - 40 + 3x + y + 19 = 180 \rightarrow (2) 3x + 11y = 201$ .  $3(1) - 4(2) = (12x - 15y = -81) + (-12x - 44y = -804) \rightarrow -59y = -885$ ,  $y = 15$ . In (1):  $4x - 5(15) = -27 \rightarrow 4x = 48$ ,  $x = 12$ .  $8x + 14 = 8(12) + 14 = 110 = m\angle QRP$ .  
 $m\angle QRT - m\angle RTS = 70 - 110 = -40$ . **Ans. - 40**
3. 17 produces 8 as sum of digits so far. The rest of the numbers have to add to 9 and be the smallest number. Thus starting with 17, then 34 etc. we are looking for multiples of 17 whose digits add to 9. The smallest is 153. The number is 15317. Multiplying the digits = **Ans. 105**

## Individuals Round 4

1. I picked 9:  $9 + 5 = 14 \times 6 = 84 - 15 = 69 / 3 = 23 - 9 = 14$ . 14 subtract 5 = 9. Students should try more than one number to prove the answer. **Ans. Subtract 5**
2. By Pyth. Thm:  $(x + 2)^2 = x^2 + (x + 1)^2 \rightarrow x^2 + 4x + 4 = x^2 + x^2 + 2x + 1 \rightarrow$   
 $0 = x^2 - 2x - 3 \rightarrow 0 = (x - 3)(x + 1)$ .  $x = 3$ . Diagonals are 6 and 8. Area =  $\frac{1}{2}(48)$ . **Ans. 24**

3.  $\frac{x^{a^2}}{x^{b^2}} = x^{28} \rightarrow x^{a^2} \cdot x^{-b^2} = x^{28}$ , thus  $a^2 - b^2 = 28$ ,  $a^2 - (4-a)^2 = 28 \rightarrow a^2 - 16 + 8a - a^2 = 28$ ,

$8a = 44$ , so  $a = 5\frac{1}{2}$  and  $b = -1\frac{1}{2}$ .  $a - b = 5\frac{1}{2} - (-1\frac{1}{2}) = 7$ . Alt. sol.:  $a^2 - b^2 = 28$ , so

$(a + b)(a - b) = 28 \rightarrow 4(a - b) = 28$ , then  $a - b = 7$ .

**Ans. 7**

### Individuals Round 5

1.  $2 * 3 = 2(3) - 3^2 = -3$ .  $4 * (-3) = 4(-3) - (-3)^4 = -12 - 81 = -93$ .

**Ans. -93**

2.  $(ax + 3)(bx + 2) = 15x^2 + cx + 6 \rightarrow abx + 3bx + 2ax + 6 = 15x^2 + cx + 6$ . Thus  $ab = 15$  and since  $a + b = 16$ , then either  $a = 15$  and  $b = 1$ , or  $a = 1$  and  $b = 15$ .  $3a + 2b = c$ , then

$c = 3(15) + 2(1) = 47$  or  $3(1) + 2(15) = 33$ .

**Ans. 47, 33**

3.  $\frac{PQ}{RS} = \frac{AE}{CD} \rightarrow \frac{m}{20} = \frac{22}{18}$ ,  $\frac{ST}{RS} = \frac{BC}{CD} \rightarrow \frac{x}{20} = \frac{24}{18}$ ,  $\frac{PT}{RS} = \frac{AB}{CD} \rightarrow \frac{y}{20} = \frac{20}{18}$ ,  $\frac{DE}{CD} = \frac{RQ}{RS} \rightarrow \frac{n}{20} = \frac{25}{18}$ .

Thus  $\frac{18}{20}(m + x + y + n) = 22 + 24 + 20 + 25 \rightarrow m + x + y + n = \frac{10}{9}(91) = \frac{910}{9} = 101\frac{1}{9}$ . This

added to 20 makes  $121\frac{1}{9}$ .

**Ans.  $121\frac{1}{9}$**

### Individuals Round 6

1. The other side has to be 12. Perimeter:  $4 + 12 + 12 = 28$ .

**Ans. 28**

2. (1)  $2x + 3y = -4$  and (2)  $2x - 3y = 8$ . (1) + (2):  $4x = 4$ ,  $x = 1$ . In (1):  $2(1) + 3y = -4$ ,

$3y = -6$ ,  $y = -2$ .  $(1, -2) = (h, k)$ ,  $h - k = 1 - (-2) = 3$ .

**Ans. 3**

3.  $\log_{13}(x^3 - 1) + \log_{\frac{1}{13}}(x - 1) = 1 \rightarrow \log_{13}(x^3 - 1) + \log_{13}\left(\frac{1}{x - 1}\right) = 1 \rightarrow \log_{13}\left(\frac{x^3 - 1}{x - 1}\right) = 1 \rightarrow$

$x^3 - 1 = 13x - 13 \rightarrow x^3 - 13x + 12 = 0 \rightarrow (x - 3)(x^2 + 3x - 4) = 0 \rightarrow (x - 3)(x + 4)(x - 1) = 0$ .

So  $x = 3, -4, 1$ .  $x$  cannot = 1 or -4.

**Ans. 3**

### Team Round 1

1.  $\frac{(xy - z)(xy + z) + z^2}{yz^3} = \frac{x^2y^2 - z^2 + z^2}{yz^3} = \frac{x^2y^2}{yz^3} = \frac{x^2y}{z^3} = x^2y^1z^{-3}$ .  $A + B - C = 6$ . **Ans. 6**

2.  $8 - \frac{n}{3} > 1 - 2(3 + n) \rightarrow 24 - n > 3 - 18 - 6n \rightarrow 5n > -39$ ,  $n > -7\frac{4}{5}$ . The negative numbers

are from -7 to -1. So  $-\frac{7(8)}{2} = -28$ .

**Ans. -28**

3. So  $2C - N =$  score. Let  $x =$  # incorrect answers. Then  $2(40 - x) - x = 50$ ,  $80 - 3x = 50 \rightarrow 30 = 3x$ , 10 incorrect answers, so 30 correct answers.

**Ans. 30**

4. 85% are non-students. 15% are student.  $.15x = 6$ ,  $x = 40$  committee members.  $45\%x - 25\%x = 18 - 10 = 8$ . **Ans. 8**

5.  $PQ^2 = PR(PU) \rightarrow 36 = 3(3 + RU) \rightarrow 12 = 3 + RU$ ,  $RU = 9$ , so  $SU = 7 = a$ .

$QS(ST) = RS(SU)$ , so  $b(4) = 2(7)$ ,  $b = 3\frac{1}{2}$ .  $4a - 2b = 4(7) - 2(3\frac{1}{2}) = 21$ . **Ans. 21**

6.  $x^y = 2^{20} = 4^{10} = 16^5 = 32^4 = 1024^2 = (20^2)^1$  **Ans. 6**

7.  $\frac{3n(3n+1)}{2} - \frac{n(n+1)}{2} = 150 \rightarrow 9n^2 + 3n - n^2 - n = 300 \rightarrow 8n^2 + 2n - 300 = 0$ .

$4n^2 + n - 150 = 0 \rightarrow (4n + 25)(n - 6) = 0$ ,  $n = 6$ .  $5n = 30$ .  $\text{Sum} = \frac{30(31)}{2} = 465$ . **Ans. 465**

8.  $\sec^2 x + 2 \cos x \sin x = 2 + \tan^2 x \rightarrow \sec^2 x - \tan^2 x + 2 \cos x \sin x = 2 \rightarrow$

$1 + \sin 2x = 2 \rightarrow \sin 2x = 1$ .  $2x = 90^\circ$ , so  $x = 45^\circ$ . Since  $\sin 2x$  repeats every  $180^\circ$ , then  $x = 225^\circ$  also. Thus  $45^\circ + 225^\circ = 270^\circ$ . **Ans. 270°**

### Team Round 2

1.  $4(a^2 - 2b + 5) - 3(4a^2 + b - 7) \rightarrow 4a^2 - 8b + 20 - 12a^2 - 3b + 21$ . Since  $a = -2b$ , then

$4(-2b)^2 - 8b + 20 - 12(-2b)^2 - 3b + 21 \rightarrow 16b^2 - 48b^2 - 11b + 41$ . **Ans.  $-32b^2 - 11b + 41$**

2.  $\frac{8th}{9th} = \frac{4}{3}$ ,  $\frac{9th}{11th} = \frac{6}{1}$ ,  $\frac{11th}{10th} = \frac{2}{3} \rightarrow \frac{4}{3} \cdot \frac{6}{1} \cdot \frac{2}{3} = \frac{16}{3}$ . **Ans. 16/3**

3.  $\frac{x^2 - 3x - 10}{x^2 + 3x - 28} \cdot \frac{x^2 + 8x + 7}{x^2 - x + 20} = 1 \rightarrow \frac{(x-5)(x+2) \cdot \cancel{(x+7)}(x+1)}{\cancel{(x+7)}(x-4) \cdot (x^2 - x + 20)} = 1 \rightarrow$

$x^3 - 2x^2 - 13x - 10 = x^3 - 5x^2 + 24x - 80 \rightarrow 3x^2 - 37x + 70 = 0 \rightarrow (3x - 7)(x - 10) = 0$ ;  $x = 7/3, 10$

**Ans. 7/3, 10**

4.  $\frac{{}_6C_2 \cdot {}_4C_1}{{}_{10}C_3} = \frac{15 \cdot 4}{10 \cdot 9 \cdot 8} = \frac{15 \cdot 4}{10 \cdot 3 \cdot 4} = \frac{1}{2}$ . **Ans. 1/2**

5. Cylinder + 2 Cones:  $\pi 5^2(10) + 2(\frac{1}{3})\pi 5^2(5) = 250\pi + \frac{2}{3}(125)\pi = \frac{1000\pi}{3}$ . **Ans.  $\frac{1000\pi}{3}$**

6. (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (3, 1),

(3, 2), (3, 3), (4, 1). There are 16.

**Ans. 16**

7. Each small triangle is an equilateral triangle with 4 unit length sides. So each triangle has an area of  $4\sqrt{3}$ . The inner hexagon can be divided into 6 congruent triangles. All 12 triangles are congruent and have the same area.  $12(4\sqrt{3}) = 48\sqrt{3}$ . **Ans.  $48\sqrt{3}$**

8.  $x(x-2)(x-3) - 150 - 3x(x+3) + 5(x-2)(x+3) + 6x^2 - 15(x-3) = -75 \rightarrow$

$x^3 - 5x^2 + 6x - 150 - 3x^2 - 9x + 5x^2 + 5x - 30 + 6x^2 - 15x + 45 = -75 \rightarrow x^3 + 3x^2 - 13x - 60 = 0 \rightarrow (x-4)(x^2 + 7x + 15)$ .  $x = 4$ , discriminant:  $\sqrt{49 - 4(15)} = \sqrt{-11}$ , no solution. **Ans. 4**

**Seat A Blue Relay**

$c = -27 + 23 = -4$ ,  $b = -4/2 + 1 = -1$ ,  $a = (-1)(-1) - (-4) = 5$ .  $A + b = 5 + (-1) = 4$ .

$2A^2 = 2(4)^2 = 32$ .

**A = 4, Pass 32**

**Seat B Blue Relay**

$25x + 100(26 - x) = 1700 \rightarrow 25x + 2600 - 100x = 1700 \rightarrow -75x = -900$ ,  $x = 12$ . 12 quarters.

That makes 3 dollars. Pass back:  $B + X + 2 = 3 + 32 + 2 = 37$ .

**B = 3, Pass 37**

**Seat C Blue Relay**

$90 - x = 8x - 54 \rightarrow 144 = 9x$ ,  $x = 16$ . Comp = 74. Pass:  $\sqrt{\frac{32C}{x}} = \sqrt{\frac{32(74)}{37}} = 8$ . **C = 74, Pass 8**

**Seat D Blue Relay**

$\log_3 6 = x$ , so  $3^x = 6$ .  $9^{2x-1} = 9^{2x}/9 = 3^{4x}/9 = (3^x)^4/9 = 6^4/9 = 144$ . Pass back:  $D - 2X =$

$144 - 2(8) = 144 - 16 = 128$ .

**D = 144, Pass 128**

**Seat E Blue Relay**

If  $\sin a = \cos b$ , then  $a$  and  $b$  are complementary.  $4x - 22 + 6x - 13 = 90 \rightarrow 10x - 35 = 90 \rightarrow$

$10x = 125 = E$ . Pass in:  $(X - E)^2 = (128 - 125)^2 = 9$ .

**E = 125, Pass 9**

**Seat A Green Relay**

$c = -25 + 19 = -6$ ,  $b = -6/2 + 1 = -2$ ,  $a = (-2)^2 + 6 = 10$ .  $a + b = 8$ .

**A = 8, Pass 105**

**Seat B Green Relay**

$10(90 - x) + 25x = 1800 \rightarrow 900 - 10x + 25x = 1800 \rightarrow 15x = 900$ ,  $\rightarrow x = 60$ . 60 quarters

makes 15 dollars. Pass back:  $X/B = 105/15 = 7$ .

**B = 15, Pass 7**

**Seat C Green Relay**

$180 - x = 7x + 36 \rightarrow 144 = 8x$ ,  $x = 18$ . Pass back:  $1000(C + X) = 25000$ . **C = 18, Pass 25,000**

**Seat D Green Relay**

$3^x = 10$ .  $27^{x+1} = 27(27)^x = 27(3^3)^x = 27(3^x)^3 = 27(10)^3 = 27,000$ . Pass back:  $\frac{D-X}{400} =$

$\frac{27000 - 25000}{400} = \frac{2000}{400} = 5$ .

**D = 27,000, Pass 5**

**Seat E Green Relay**

$7k - 21 + 4k + 12 = 90, 11k = 99, x = 9.$  Pass in:  $X^{(E-X)} = 5^4 = 625.$  **E = 9, Pass 625**

**Seat A Pink Relay**

$\sqrt{605} = 11\sqrt{5}. 5\sqrt{11} = \sqrt{c}, c = 25(11) = 275.$  Pass:  $A - 19 = 256.$  **A = 275, Pass 256**

**Seat B Pink Relay**

$\frac{x-4}{27-x+5} = \frac{1}{3} \rightarrow 3x - 12 = 32 - x \rightarrow 4x = 44, x = 11.$  Orig. Frac. =  $\frac{11}{16}.$

Pass back:  $BX - 66 = (\frac{11}{16})(256) - 66 = 11(16) - 66 = 176 - 66 = 110.$  **B = 11/16, Pass 110**

**Seat C Pink Relay**

$DC = 6, BD = 6\sqrt{3}. (6\sqrt{3})^2 = 36(3) = 108.$  Pass back:  $(110 - 108)^5 = 32.$  **A = 108, Pass 32**

**Seat D Pink Relay**

x-coordinate of vertex is at  $\frac{-b}{2a} = \frac{-(-8)}{2(2)} = 2.$  Vertex:  $(2, 24).$  Plugging in:  $2(2)^2 - 8(2) + k = 24.$

$8 - 16 + k = 24, k = 32.$  Pass back:  $2\sqrt{DX} = 2\sqrt{32(32)} = 64.$  **A = 32, Pass 64**

**Seat E Pink Relay**

$\begin{bmatrix} \frac{x}{2} & 1 & 1 \\ 2 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} \frac{x}{2} + 2 + 2 & x + 3 - 2 \\ 2 + 4 + 6 & 4 + 6 - 6 \end{bmatrix} = \begin{bmatrix} \frac{x}{2} + 4 & x + 1 \\ 12 & 4 \end{bmatrix} = -16, 2x + 16 - 12x - 12 = -16 \rightarrow$

$-10x = -20, x = 2.$  Pass in:  $7\sqrt[5]{X} = 7\sqrt[5]{64} = 56.$  **A = 2, Pass 56**

**Seat A Yellow Relay**

$\sqrt{245} = 7\sqrt{5}. 5\sqrt{7} = \sqrt{c},$  so  $c = 175.$  Pass back:  $A - 15 = 160.$  **A = 175, Pass 160**

**Seat B Yellow Relay**

$\frac{x+5}{47-x-2} = \frac{2}{3} \rightarrow 3x + 15 = 90 - 2x \rightarrow 5x = 75, x = 15.$  Orig. frac. =  $\frac{15}{32} = B.$

Pass back:  $BX = \frac{15}{32}(160) = 75.$  **B = 15/32, Pass 75**

### Seat C Yellow Relay

Since  $BD = 6\sqrt{3}$ , then  $DC = 6$  and  $BC = 12$ . Thus perimeter = 36. Pass back:

$$\left(\frac{X}{\sqrt{C}+9}\right)^2 = \left(\frac{75}{\sqrt{36}+9}\right)^2 = \left(\frac{75}{15}\right)^2 = 5^2 = 25.$$

**C = 36, Pass 25**

### Seat D Yellow Relay

Same as Pink C:  $\frac{-b}{2a} = \frac{-8}{2(2)} = -2$ .  $2(-2)^2 + 8(-2) + k = -8 \rightarrow 8 - 16 + k = -8$ ,  $k = 0 = D$ .

Pass back:  $D + X + 2 = 0 + 25 + 2 = 27$ .

**D = 0, Pass 27**

### Seat E Yellow Relay

$$\begin{bmatrix} \frac{x}{2} & 1 & 3 \\ -3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} x+3-6 & \frac{x}{2}+2+6 \\ -6+6-2 & -3+4+2 \end{bmatrix} = \begin{bmatrix} x-3 & \frac{x}{2}+8 \\ -2 & 3 \end{bmatrix} = 23 \rightarrow 3x - 9 + x + 16 = 23 \rightarrow$$

$$4x + 7 = 23 \rightarrow 4x = 16, x = 4.$$

$$\text{Pass in: } \left(\frac{X}{E-1}\right)^4 = \left(\frac{27}{4-1}\right)^4 = 9^4 = 6561.$$

**E = 4, Pass 6561**

## Answer Sheet – States 2019

### Individuals Round 1

1.  $6x^3 - 2x^2 + 3x - 2$
2.  $-5$
3. 2025

### Individuals Round 2

1.  $-2010$
2. 57
3.  $25/4$  or  $6\frac{1}{4}$  or 6.25

### Individuals Round 3

1. 100
2.  $-40$

3. 105

### Individuals Round 4

1. subtract, 5
2. 24
3. 7

### Individuals Round 5

1.  $-93$
2. 33, 47
3.  $121\frac{1}{9}$  or  $\frac{1090}{9}$

### Individuals Round 6

1. 28
2. 3
3. 3

### Team Round 1

1. 6
2.  $-28$
3. 30
4. 8
5. 21
6. 6

7. 465
8.  $270^o$

### Team Round 2

1.  $-32b^2 - 11b + 41$
2. 16:3 or  $16/3$
3.  $7/3, 10$

4.  $1/2$
5.  $\frac{1000\pi}{3}$
6. 16
7.  $48\sqrt{3}$
8. 4

### Blue Relay

- A = 4, Pass 32  
 B = 3, Pass 37  
 C = 74, Pass 8  
 D = 144, Pass 128  
 E = 125, Pass 9

### Green Relay

- A = 8, Pass 105  
 B = 15, Pass 7  
 C = 18, Pass 25,000  
 D = 27,000, Pass 5  
 E = 9, Pass 625

### Pink Relay

- A = 275, Pass 256  
 B =  $11/16$ , Pass 110  
 C = 108, Pass 32  
 D = 32, Pass 64  
 E = 2, Pass 56

### Yellow Relay

- A = 175, Pass 160  
 B =  $15/32$ , Pass 75  
 C = 36, Pass 25  
 D = 0, Pass 27  
 E = 4, Pass 6561