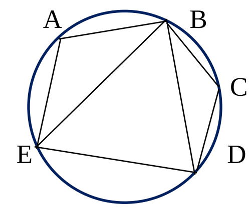
Individuals Round 1 States 2018

3 pts 1. Welcome to the Maine State Math Meet! At today's meet there are 6 individual rounds, each with a 3-point, a 4-point and a 5-point problem. An individual's score for the meet is the total of all points scored during these 6 rounds. How many unique individual score totals are there?

Ans. ________________________

4 pts 2. ABCDE is a pentagon inscribed in a circle. M$\angle$EAB = 110° and m$\angle$BCD = 120°. Find the absolute value of the difference between the degree measure of the largest angle and the smallest angle in triangle BDE. **Ans.** ________________________



5 pts 3. Just out of curiosity, how many positive integer factors does 15! have?

Ans. ________________________

Individuals Round 2 States 2018

3 pts 1. In how many ways can 7 students line up to leave a classroom, if the girls must leave ahead of the 4 boys?

Ans. ________________________

4 pts 2. Buses leave Boston going north on I-95 to Maine every hour on the hour. If the buses all travel at 55 mph, then a limousine traveling at 70 mph south on I-95 from Maine will pass a northbound bus every *N* minutes. Find *N* to nearest tenth.

Ans. ________________________

5 pts 3. Find the sum of all values of θ, where $0^\circ \le \theta < 360^\circ$, that satisfies the following:

$$3\tan\theta\sec^2\theta - 4\tan\theta = 3\sqrt{3}\sec^2\theta - 4\sqrt{3}$$

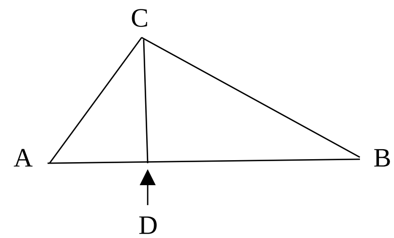
Ans. ________________________

Individuals Round 3 States 2018

3 pts 1. Suzie's bicycle has tires with a radius of 20 inches. Each of her tires makes 780 full revolutions when she rides to the gym. How many revolutions would the tires each make, if she instead rode her brother's bicycle that has 24 inch radius tires?

Ans. ___________________________

4 pts 2. In the right triangle ABC, angle C is right angle. $\overline{CD}$ is drawn perpendicular to $\overline{AB}$ at D. If AD = 10 and BC = 12, find the length of $\overline{CD}$.



Ans. ___________________________

5 pts 3. Teams A and B play in a local baseball league. At one point, team A had won $62\frac{1}{2}\%$ of its games and team B had won $58\frac{1}{3}\%$ of its games. If team B had won 12 more games than team A and also lost 12 more games than team A, how many games had team A lost?

Ans. ____________________

Individuals Round 4 States 2018

3 pts 1. The first three terms of an arithmetic progression are $\frac{7}{32}, \frac{17}{64}$ and $\frac{5}{16}$. Find the 21st term. Express as an improper fraction in simplest form.

Ans. ____________________

4 pts 2. Find the least possible positive value of constant *k*, so that each member of the set $\left\{\frac{25}{12}k, \frac{35}{48}k, \frac{75}{108}k\right\}$ is an integer. Express *k* as an improper fraction in simplest form.

Ans. ____________________

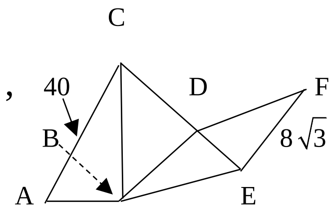
5 pts 3. The elements in data set A have a mean value of 8. The elements in data set B have a mean of 25. If the mean value of the elements in the union of A and B is 19 and both A and B have at least 100 elements, what is the least possible number of elements in set B?

Ans. _____________________

Individuals Round 5 States 2018

3 pts 1. Find integer N, if $\log_4 N = 3 + \log_4 3$. **Ans.** ______________________

4 pts 2. If AC = 40, FE = $8\sqrt{3}$, m$\angle$BAC = 60º = m$\angle$BED, $\overline{AB} \perp \overline{BC}$, $\overline{BD} \perp \overline{CE}$, $\overline{EF} \perp \overline{CE}$ and $\overline{CD} \cong \overline{BD}$. Find DF.



Ans. ___________________________

5 pts 3. Let f(x) = $\frac{k}{1-x}$. Find all positive values of constant k such that f^{-1}(x) will equal x for some real value of x.

Ans. _________________________

Individuals Round 6 States 2018

3 pts 1. Find xy if 22x + 23y = 16 and 23x + 22y = 29. **Ans.** ____________________

4 pts 2. Two lines, whose slopes are 2/3 and 5/8, have an x-intercept of 6. What is the absolute value of the difference of their y-intercepts? Give answer as a fraction.

Ans. ______________________

5 pts 3. A lookout is positioned on a crow's nest of a ship, and the lookout's eyes are 52.8 feet above the surface of the ocean. To the nearest integer, how far from the ship in miles can the lookout see the horizon? Assume the earth is a perfect sphere with radius of 4000 miles.

Ans. _____________________

Team Round 1 States 2018

4 pts 1. Sally receives an electronic payment every month "on the third Wednesday of the month". The dates of the days within the months on which she receives these payments are therefore from day number A (earliest day of month) through day B (latest day), inclusive. Find the product AB.

(1) Ans. ____________________ 4 pts

4 pts 2. Find $|x-y|$ if $\sqrt{x}$ + $\sqrt{y}$ = $\sqrt{7+2\sqrt{10}}$. **(2) Ans. ____________________ 4 pts**

6 pts 3. The circle $x^2 + y^2 - 2x + 4y - 164 = 0$ intersects the line $x = -4$ at two points, $(-4, y_1)$ and $(-4, y_2)$. Find the product of y_1 and y_2.

(3) Ans. ____________________ 6 pts

6 pts 4. Let *N* equal the number of unique non-empty subsets of the set S, where S = {A, B, C, D, E, F, G, H}. Find the value of *N* in the binary system of numeration (N is in Base 2). Remember that any set is a subset of itself.

(4) Ans. __________________________ 6 pts

6 pts 5. The 9 ordered triples listed below are all side lengths of triangles. How many are right triangles? (8, 15, 17), (18, 80, 82), (65, 72, 97), (37, 51, 63), (30, 40, 50), $\left(\frac{1}{2},\frac{2}{3},\frac{5}{6}\right)$,

(24, 70, 74), (19, 77, 80), (15, 36, 39) **(5) Ans. _______________________ 6 pts**

8 pts 6. Find the greatest prime number factor of 124,488, which equals $(53^3 - 29^3)$.

(6) Ans. ________________________ 8 pts

8 pts 7. April and May both begin running laps at the start/finish line of a circular track at the same time, but run in opposite directions. If both runners maintain constant speeds and they first pass each other when April has gone 170 degrees around the track, how many laps has May run when the two first pass each other at the start/finish line?

(7) Ans. _____________________ 8 pts

8 pts 8. A number is chosen at random from the numbers 200 to 700. What is the probability that the number chosen is divisible by 2 or 3, but not 6? Express as a fraction in simplest form.

(8) Ans. ____________________ 8 pts

Team Round 2 States 2018

4 pts 1. Sylvia has 6 quarters in her pocket, including 2 rare quarters. If she selects 2 of the quarters at random, what is the probability she selects neither of the rare coins? Write answer as a fraction.

(1) Ans. ___________4 pts

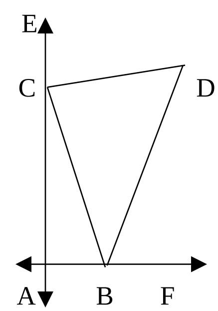
4 pts 2. For how many integer values of x is $-20 < \sqrt[3]{x} < -10$? **(2) Ans. ______________ 4 pts**

6 pts 3. If $\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & 3 \\ 6 & 5 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -16 \\ 34 \end{bmatrix}$, find the value of x + y – z. **(3) Ans. ______________ 6 pts**

6 pts 4. Square ABCD is inscribed in a circle. The circle is inscribed in a 12-35-37 right triangle. Point E is selected at random on $\overline{AB}$ and triangle ECD is drawn. Find the area of triangle ECD.

(4) Ans. _________________________6 pts

6 pts 5. On the Cartesian coordinate system at right, C is on the positive y-axis and B is on the positive x-axis. $\overline{CD}$ bisects $\angle$BCE, and $\overline{BD}$ bisects $\angle$CBF. Find m $\angle$D in degrees.



(5) Ans. __________________ 6 pts

8 pts 6. The points P = $(1 + i)^{10}$ and Q = $(1 – i)^{10}$ are graphed in the complex plane. Find the distance in this plane between P and Q.

(6) Ans. _________________________ 8 pts

8 pts 7. Let *p*(x) = $x^4 + ax^3 + bx^2 + cx + d$, where *a*, *b*, *c* and *d* are integers. If there exists 40 possible unique real roots of p(x), for a given value of *d* with unknown integer values for *a*, *b*, and *c*, find the least possible value for *d*.

(7) Ans. _____________________ 8 pts

8 pts 8. Ignoring the colon, the display on a 12 hour digital clock can read as a three- or four-digit whole number. How many of the whole numbers displayed are perfect squares?

(8) Ans. _________________ 8 pts

Blue Seat A States 2018

Find y given the system: $7x - 12y = -19$ and $5x + 6y = 103$.

Pass Back: A^2 A = Your Answer.

Blue Seat B States 2018

Runners R and S both leave a common starting point at the same time. R runs at 7 mph and S runs at 9 mph. When S reaches a house 7 miles from the start, she immediately turns around and runs back toward the starting point. How many miles from the house do R and S pass each other?

Pass back: 5BX B = Your answer. X = The number you will receive.

Blue Seat C States 2018

One chord in a circle has endpoints (1, 7) and (8, 7), while another chord has endpoints (0, 1) and (4, 9). Find the y-coordinate of the center of the circle. Express as mixed number.

Pass back: CX C = Your answer. X = The number you will receive.

Blue Seat D States 2018

The expression $x^3 - 15x^2 + 67x - 117$ factors into $(x - [3 + 2i])(x - [3 - 2i])(x - k)$. Find the value of k. $i = \sqrt{-1}$

Pass back: $\frac{X}{D+21}$ D = Your answer. X = The number you will receive.

Blue Seat E States 2018

Evaluate: $\sin\left(\tan^{-1}\left(\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)\right)\right)$, using only first quadrant angles.

Pass in: E^2X E = Your answer. X = The number you will receive.

Green Seat A States 2018

Find y given that: $9x - 8y = -151$ and $-7x + 4y = 93$

Pass back: $(A + 2)^2$ A = Your answer.

Green Seat B States 2018

Runners R and S both leave a common starting point at the same time. R runs at 6 mph and S runs at 7 mph. When S reaches a house 10 miles from the start, she immediately turns around and runs back toward the starting point. How many miles from the house do R and S pass each other?

Pass back: $\frac{4BX}{5}$ B = Your answer X = The number you will receive.

Green Seat C States 2018

One chord in a circle has endpoints (1, 7) and (8, 7), while another chord has endpoints (-1, 2) and (5, 10). Find the y-coordinate of the center of the circle. Express as mixed number.

Pass back: CX C = Your answer. X = The number you will receive.

Green Seat D States 2018

The expression $x^3 - 17x^2 + 65x - 169$ factors into $(x - [2 + 3i])(x - [2 - 3i])(x - k)$. Find the value of k.

Pass back: $\frac{X}{D}$ D = Your answer. X = The number you will receive.

Green Seat E States 2018

Evaluate: $\sec\left(\cot^{-1}\left(\sin\left(\cos^{-1}\left(\frac{1}{2}\right)\right)\right)\right)$, using only first quadrant angles.

Pass in: E^2X E = Your answer. X = The number you will receive.

Pink Seat A States 2018

If J is 90% of K, what percent is $2J$ of $3K$? Answer is A%, find A.

Pass back: $\frac{3}{5}$A   A = Your answer.

Pink Seat B States 2018

Molly buys 2 boxes of candy and gets 126 cents in change, when she pays with a ten dollar bill. How many cents of change would she receive if she bought 11 boxes and paid with a fifty dollar bill? Assume no sales tax and each box is priced the same.

Pass back: B – 5X   B = Your answer.   X = The number you will receive.

Pink Seat C States 2018

In triangle ABC, AB = 14, BC = 15 and CA = 21. Point D is placed on $\overline{BC}$ and segment $\overline{AD}$ is drawn. If $\overline{AD}$ bisects $\angle$BAC, find the length DC.

Pass back: $\dfrac{3C+5X}{2}$ C = Your Answer.   X = The number you will receive.

Pink Seat D States 2018

If the interval $4\frac{3}{4} \le x \le 14\frac{1}{8}$ is divided into sub-intervals with lengths from the smaller number to the larger number in the ratio of 3:4:5:6:7, where is the division between the 3rd and 4th sub-intervals? Give answer as mixed number.

Pass back: $\dfrac{4D+9}{X}$ D = Your Answer.   X = The number you will receive.

Pink Seat E States 2018

P is a perfect number if and only if there exists some positive integer, N, such that

P = $(2^N - 1)(2^{N-1})$ and $2^N - 1$ is a prime number. Find the sum of the least three perfect numbers.

Pass in: $\sqrt{E-X}$ E = Your Answer.   X = The number you will receive.

Yellow Seat A States 2018

If J is 30% of K, what percent is $3J$ of $2K$? Answer is A%, find A.

Pass back: $\frac{3}{5}$A   A = Your answer.

Yellow Seat B States 2018

Molly buys 3 boxes of candy and gets 76 cents in change, when she pays with a ten dollar bill. How many cents of change would she receive if she bought 16 boxes and paid with a fifty dollar bill? Assume no sales tax and each box is priced the same.

Pass back: 3X – B   B = Your answer.   X = The number you will receive.

Yellow Seat C States 2018

In triangle ABC, AB = 15, BC = 24 and CA = 25. Point D is placed on $\overline{BC}$ and segment $\overline{AD}$ is drawn. If $\overline{AD}$ bisects $\angle$BAC, find the length DC.

Pass back: $\frac{3C+5X}{9}$ C = Your Answer.   X = The number you will receive.

Yellow Seat D States 2018

If the interval $4\frac{1}{4} \leq x \leq 16\frac{1}{8}$ is divided into sub-intervals with lengths from the smaller number to the larger number in the ratio of 4:5:6:7:8, where is the division between the 2^{nd} and 3^{rd} sub-intervals? Give answer as mixed number.

Pass back: $\frac{100X}{D}$ D = Your Answer.   X = The number you will receive.

Yellow Seat E States 2018

P is a perfect number if and only if there exists some positive integer, N, such that

P = $(2^N - 1)(2^{N-1})$ and $2^N - 1$ is a prime number. Find the fourth least perfect number.

Pass in: $\sqrt{\frac{E+64}{X}}$ E = Your Answer.   X = The number you will receive.

Solutions – Individuals Round 1

1. The range of integers is from 0 to 72, 73 integers. A student cannot get a 1, 2, 70 or 71.

73 – 4 = 69. **Ans. 69**

2. In quadrilateral ABDE, if A = 110°, then BDE = 70°, since opposite angles of a quadrilateral inscribed in a circle are supplementary. Likewise in quadrilateral BCDE, if C = 120°, then BED is 60°. Thus angle EBD = 50°. 70 – 50 = 20. **Ans. 20**

3. $15! = 15\cdot14\cdot13\cdot12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2$. Writing the prime factors each with the proper power from this product: $13\cdot11\cdot7^2\cdot5^3\cdot3^6\cdot2^{11}$. Raising each power by one and multiplying:

$2\cdot2\cdot3\cdot4\cdot7\cdot12$ = 144(28) = 4032. **Ans. 4032**

Individuals Round 2

1. The girls can lie up 3! = 6 ways. The boys can 4! = 24 ways. 6(24) = 144. **Ans. 144**

2. We need to know the time it takes at 125 m/h to cover 55 miles. 55/125 = 11/25 hr.

$\frac{11}{25}(60) = \frac{11\cdot12}{5} = \frac{132}{5} = 26.4$ **Ans. 26.4**

3. $3\tan\theta\sec^2\theta - 4\tan\theta = 3\sqrt{3}\sec^2\theta - 4\sqrt{3} = 0 \Rightarrow$

$3\sec^2\theta(\tan\theta - \sqrt{3}) - 4(\tan\theta - \sqrt{3}) = 0 \Rightarrow (3\sec^2\theta - 4)(\tan\theta - \sqrt{3}) = 0$.

Either (1): $\tan\theta = \sqrt{3}$ or (2): $3\sec^2\theta - 4 = 0$. In (1) θ = 60° or 240°. In (2): $\sec^2\theta = \frac{4}{3}$,

$\sec\theta = \pm\frac{2}{\sqrt{3}}$. θ = 30°, 150°, 210° or 330°. The sum of all 6 is 1020°. **Ans. 1020°**

Individuals Round 3

1. The distance to the gym = $2\pi(20)(780)$. Divide this by $2\pi(24)$ = $\frac{5}{6}(780)$ = 650. **Ans. 650**

2. All 3 are similar rt Δ's. So $\frac{BD}{CB} = \frac{CB}{AB}$. Let BD = x, then $\frac{x}{12} = \frac{12}{x+10} \Rightarrow x^2 + 10x - 144 = 0$.

Factoring (x – 8)(x + 18) = 0. So x = 8 = AD. Now $\frac{AD}{CD} = \frac{CD}{BD} \Rightarrow \frac{10}{CD} = \frac{CD}{8} \Rightarrow$ CD = **Ans.** $\mathbf{4\sqrt{5}}$

3. Note that 62 ½ % = 5/8 and 58 1/3 % = 7/12. Suppose that team A has 5x wins and thus 3x losses for some positive integer x. Then team B has 5x + 12 wins and 3x + 12 losses. Thus we have $\frac{5x+12}{8x+24} = \frac{7}{12} \Rightarrow$ 60x + 144 = 56x + 168 $\Rightarrow$ 4x = 24, so x = 6. A has 3(6) losses. **Ans. 18**

Individuals Round 4

1. The common difference = $\frac{17}{64}-\frac{7}{32}=\frac{17-14}{64}=\frac{3}{64}$. 21[st] term: $\frac{7}{32}+20\left(\frac{3}{64}\right)$ = $\frac{14}{64}+\frac{60}{64}$ **Ans.** $\frac{37}{32}$

2. To begin with $\frac{75}{108}k$ reduces to $\frac{25}{36}k$. Now we need $\frac{LCM\{12,48,36\}}{GCF\{25,35\}}$ = $\frac{144}{5}$. **Ans.** $\frac{144}{5}$

3. Let a and b equal the number of elements in sets A and B, respectively. Then 8a is the sum of the elements of A, and 25b is the sum of the elements of B. Then $\frac{8a+25b}{a+b}=19$ or

8a + 25b = 19a + 19b ➔ 6b = 11a and $\frac{a}{b}=\frac{6}{11}$. $\frac{a}{b}=\frac{6k}{11k}$ for some number *k*. Since there are at least 100 elements in each set, the 6k $\geq$ 100, so the least value of k is 17, and set B must have at least 11*k* = 11(17) = 187. **Ans. 187**

Individuals Round 5

1. $\log_4 N = 3+\log_4 3$ ➔ $\log_4 N-\log_4 3=3$ ➔ $\log_4\frac{N}{3}$ = 3 ➔ $\frac{N}{3}$ = 64, so *N* = 192. **Ans. 192**

2. From AC to DE ➔ AB to AB to BC to BD to DE =$\frac{40\cdot\sqrt{3}}{2\cdot\sqrt{2}\cdot\sqrt{3}}$ = $\frac{20}{\sqrt{2}}$ = $10\sqrt{2}$.

DF = $\sqrt{\left(10\sqrt{2}\right)^2+\left(8\sqrt{3}\right)^2}$ = $\sqrt{200+192}$ = $\sqrt{392}$ = $\sqrt{4\cdot 49\cdot 2}$ = $14\sqrt{2}$. **Ans. $14\sqrt{2}$**

3. f(x) = $\frac{k}{1-x}$. Let f(x) = y, now switch y and x: $x=\frac{k}{1-y}$. Now solving for y will give us $f^{-1}(x)$: x(1 – y) = k ➔ x – xy = *k* ➔ -xy = k – x, y = $\frac{x-k}{x}$ = $f^{-1}(x)$. Thus $\frac{x-k}{x}$ = x ➔

x – *k* = x^2 ➔ x^2 - x + *k* = 0. x = $\frac{1\pm\sqrt{1-4k}}{2}$. In order for x to be real and k to be positive, then 1 – 4*k* $\geq$ 0. 1 $\geq$ 4k, so 1/4 $\geq$ *k*. **Ans. 0 < *k* $\leq$ 1/4**

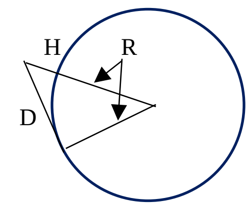
Individuals Round 6

1. (1) 22x + 23y = 16 and (2) 23x + 22y = 29. Adding (1) and (2): 45x + 45y = 45 or x + y = 1. Subtracting (2) – (1): x - y = 13. Add the new ones: 2x = 14, so x = 7, y = -6. 7(-6) **Ans. -42**

2. Line m: y = $\frac{2}{3}$x + b ➔ 3y = 2x + 3b ➔ 2x = 3y – 3b ➔ x = $\frac{3}{2}$ y – $\frac{3}{2}$b. x-intercept = – $\frac{3}{2}$b.

Line n; y = $\frac{5}{8}$x + a ➔ 8y = 5x + 8a ➔ 5x = 8y – 8a ➔ x = $\frac{8}{5}$y - $\frac{8}{5}$a. x-intercept = - $\frac{8}{5}$a. For line m: $-\frac{3}{2}b = 6$, $b = 6\left(-\frac{2}{3}\right) = -4$. For line n: $-\frac{8}{5}a = 6$, $a = 6\left(-\frac{5}{8}\right) = -3\frac{3}{4}$. $\left|-4-\left(-3\frac{3}{4}\right)\right|$ **Ans.** $\frac{1}{4}$

3. Using figure at right: Let R = radius of earth, H = Height over the water, and D = the distance to the horizon. $R^2 + D^2 = (R + H)^2$ (Pyth. Thm)



$D^2 = 2RH + H^2$, so $D = \sqrt{2RH + H^2}$. Converting H from ft to mi: $\frac{52.8}{5280} = \frac{1}{100}$. Therefore

$D = \sqrt{2(4000)(.01) - (.01)^2} = \sqrt{80 + .0001} = 9$ to the nearest mile. **Ans. 9**

Team Round 1

1. The first possible Wednesday would be 14 days after the first possible starting day which would be a Wednesday the first. That day would be the 15th, and the last possible Wednesday would fall on the 21st. 15(21) = 315. **Ans. 315**

2. Squaring $\sqrt{x} + \sqrt{y} = \sqrt{7+2\sqrt{10}}$, yields $x + y + 2\sqrt{xy} = 7 + 2\sqrt{10}$. Thus x + y = 7 and xy = 10. x(7 – x) = 10 ➔ $x^2 - 7x + 10 = 0$ ➔ (x – 2)(x – 5) = 0. x = 2 and y = 5 or vice-versa. Either way the absolute value of the difference is 3. **Ans. 3**

3. Plugging -4 for x into $x^2 + y^2 - 2x + 4y - 164 = 0$ yields $16 + y^2 + 8 = 4y - 164 = 0$ ➔

$y^2 + 4y - 140 = 0$. The product of these two solutions is -140. **Ans. -140**

4. The number of subsets of a set with n elements is 2^n. $2^8 = 256$. Since this includes the empty set and we want "non-empty" subsets, then there are 255. Converted to the binary system, this would be 11,111,111. **Ans. 11,111,111**

5. Some of these are the basic Pythagorean triplets used by some students. 1 is, 2 is a 9-40-41 multiple, 5 and 6 are 3-4-5 triangles, 24, 70, 74 is a 12-35-37 multiple, which will be used in the second team round, and the last one is a 5-12-13 multiple. That makes 6. Students will have to multiply for 3, 4, and 7. For 3: $4225 + 5184 = 9409 = 97^2$; for 4: the unit's digits do not match 9 + 1 = 9, not. For 8: unit's digits do not match again 9 + 9 = 0, not. **Ans. 7**

6. $53^3 - 29^3 = (53 - 29)$ (whatever). 53 – 29 = 24, which is a factor. 124,488/24 = 5187, which is a multiple of 3. 5187/3 = 1729. 1729/7 = 247. 247/13 = 13(19). **Ans. 19**

7. The passing points will move 170° in April's direction each time. They will reach the start/finish line when April has run 170k degrees, where k is the least positive integer such that 170k is a multiple of 360, or that 17k is a multiple of 36. Since 17 and 36 are relatively prime, k = 36. So April runs a total of 36(170) degrees or 17 laps. Since May runs 19/17 as fast, she runs 19 laps. **Ans. 19**

8. From 200 to 700 is 700 – 200 +1 = 501 integers. Multiples of 2: 700 = 200 + (n – 1) 2 ➔ 350 = 100 + (n – 1) = 251. Multiples of 3: 699 = 201 + (n -1)3 ➔ 233 = 67 + (n – 1) = 167. Each of these contain multiples of 6. Mult. Of 6: 696 = 204 + (n – 1) 6 ➔ 116 = 34 + (n – 1) = 83. Numerator = 251 + 167 – 2(83) = 252. 252/501 = 84/167. **Ans. 84/167**

Team Round 2 States 2018

1. There are $\binom{6}{2}$ possible = 15, and $\binom{4}{2}$ favorable = 6. $\frac{6}{15} = \frac{2}{5}$. **Ans. 2/5**

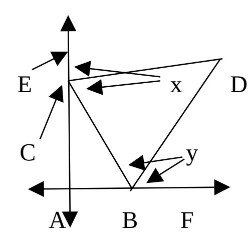
2. Cubing $-20 < \sqrt[3]{x} < -10$ ➔ -8000 < x < -1000. -1001 – (-7999) + 1 = 6999. **Ans. 6999**

3. (1) 2x + 3y – z = 14, (2) x – y + 3z = -16, (3) 6x + 5y – 7z = 34.

(1) – 2(2) ➔ (4): 5y – 7z = 46; (3) – 6(2) ➔ (5): 11y – 25z = 130. 11(4) – 5(5) ➔ 48z = -144. So z = -3. In (4): 5y – 7(-3) = 46, 5y = 25, y = 5. In (1); 2x + 3(5) – (-3) = 14. x = -2.

x + y – z = -2 + 5 – (-3) = 6. Alt. Sol.: Add (1), (3): 8x + 8y – 8z = 48, so x + y – z = 6. **Ans. 6**

4. The area of the 12-35-37 triangle is $\frac{1}{2}(12)(35) = 6(35)$. The area of the triangle could have been gotten by $\frac{1}{2}$ap, where a is the radius of the circle: 6(35) = $\frac{1}{2}$a(84), so a = $\frac{6 \cdot 35}{42}$ = 5. So the diagonal of the square is 10, the area of the square is 50, and a$\triangle$ECD = 25. **Ans. 25**



5. Let m$\angle$ECD = x = m$\angle$DCB, m$\angle$CBD = y = m$\angle$DBF, m$\angle$ABC = n and m$\angle$ACB = m. Then m + 2x = 180 and n + 2y = 180. Thus m + n + 2x + 2y = 360. Since m + n = 90, Then 2x + 2y = 270, or x + y = 135. Thus m$\angle$D = 45. **Ans. 45**

6. P = $(1 + i)^{10} = ((1 + i)^2)^5 = (1 + 2i + i^2)^5 = (2i)^5 = 32i^5 = -32i$.

Q = $(1 – i)^{10} = ((1 – i)^2)^5 = (1 – 2i + i^2)^5 = (-2i)^5 = -32i^5 = 32i$. Each is 32 from 0. **Ans. 64**

7. If there are 40 unique roots 20 are positive and 20 negative. For factors, we can have p^{19}, p^9q, p^4q^3, or p^4qr. P would need to be the least prime = 2, the next two least primes would be 3 and 5. Producing the least for d: $2^4(3)(5) = 16(15) = 240$. **Ans. 240**

8. This is a teamwork problem: 100, 121, 144, 225, 256, 324, 400, 441, 529, 625, 729, 841, 900, 1024, 1156, 1225. **Ans. 16**

Blue Relay Seat A

2(5x + 6y = 103) + (7x – 127 = - 19) ➔ 17x = 187, so x = 11 and y = 8. **Ans. 8, Pass: 64**

Blue Relay Seat B

Total distance covered by the time they meet 14 miles. 7T + 9T = 14, T = $\frac{14}{16}=\frac{7}{8}$. $7\left(\frac{7}{8}\right)$ = $6\frac{1}{8}$.

If R runs 6 1/8 miles when she meets S, she is 7/8 miles from the house.

Pass: 5BX = 5(7/8)64 = 280. **Ans. B = 7/8, Pass 280**

Blue Relay Seat C

The perpendicular bisectors of two distinct non-parallel chords intersect at the center of the circle. The perpendicular bisector of the first line is x = 9/2. The second is y = - ½ x + 6.

These two lines meet at x = 9/2, y = 15/4. Pass: CX = $\frac{15}{4}$(280) = 1050. **C = 3$\frac{3}{4}$, Pass 1050**

Blue Relay Seat D

Multiplying the first two factors produces x^2 – 6x + 13. Dividing the polynomial by this produces x – 9. Thus k = 9. – OR- Multiplying 3 $\pm$ 2i, yields 13. Since the product of the roots of a 3rd degree polynomial is the negative of its constant, or –(-117) = 117. 117/13 = 9.

Pass: $\frac{X}{D+21}$ = $\frac{1050}{9+21}$ = $\frac{1050}{30}$ = 35. **Ans. D = 9, Pass 35**

Blue Relay Seat E

$\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$ = $\frac{\sqrt{3}}{2}$. $\sin\left(\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ = $\frac{\sqrt{3}}{\sqrt{7}}$, since $\sqrt{3}$ is the opposite side and $\sqrt{\left(\sqrt{3}\right)^2+2^2}$ =

$\sqrt{7}$ is the hypotenuse. Pass: E^2X = $\left(\frac{\sqrt{3}}{\sqrt{7}}\right)^2$(35) = 15. **Ans. E = $\sqrt{21}$/7, Pass: 15**

Green Relay Seat A

(9x – 8y = -151) + 2(-7x + 4y = 93) ➔ - 5x = 35, so x = -7, y = 11. Pass: $(11 + 2)^2$ = 169

Ans. A = 11, Pass 169

Green Relay Seat B

Total distance covered by the time they meet 20 miles. 7T + 6T = 20, T = $\frac{20}{13}$. $6\left(\frac{20}{13}\right)$ = $9\frac{3}{13}$.

If R runs 9 3/13 miles when she meets S, she is 10/13 miles from the house.

Pass: $\frac{4BX}{5}=\frac{4\left(\frac{10}{13}\right)169}{5}=8\cdot 13$ = 104. **Ans. B = 10/13, Pass 104**

Green Relay Seat C

As in Blue C: x = 9/2 for one bisector, y = $-\frac{3}{4}x + \frac{15}{2}$. Intersection: $\left(\frac{9}{2}, \frac{33}{8}\right)$. So C = 33/8.

Pass: CX = (33/8)(104 = 33(13) = 429. **Ans. C = $4\frac{1}{8}$, Pass 429**

Green Relay Seat D

Same as Blue D: $x^2 - 4x + 13$ divides $x^3 - 17x^2 + 65x - 169$ ➔ x – 13, so x = 13.
Pass: X/D = 429/13 = 33. **Ans. D = 13, Pass 33**

Green Relay Seat E

$\sin\left(\cos^{-1}\left(\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{2}$. $\sec\left(\cot^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) = \frac{\sqrt{7}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$. Pass: $E^2X = \left(\frac{\sqrt{7}}{\sqrt{3}}\right)^2(33) = 77$.

Ans. E = $\sqrt{21}$/3, Pass 77

Pink Relay Seat A

J = .9K, so $\frac{J}{K} = \frac{9}{10}$. $\frac{2J}{3K} = \frac{9\cdot 2}{10\cdot 3} = \frac{3}{5}$ = 60%. Pass: $\frac{3}{5}(60) = 36$. **Ans. A = 60, Pass 36**

Pink Relay Seat B

Working only with cents: 1000 – 126 = 874. 874/2 = 437 (cost per box). 11(437) = 4807.
5000 – 4807 = 193. Pass: B – 5X = 193 – 5(36) = 193-180 = 13. **Ans. B = 193, Pass 13**

Pink Relay Seat C

The angle bisector of a triangle splits the opposite side into lengths proportional to the other two sides of the triangle. Thus $\frac{AB}{AC} = \frac{BD}{DC}$ ➔ $\frac{14}{21} = \frac{2}{3} = \frac{15-DC}{CD}$ ➔ 2DC = 45 -3DC ➔ 5DC = 45, so DC = 9. Pass: $\frac{3C+5X}{2} = \frac{3(9)+5(13)}{2} = \frac{27+65}{2} = 46$. **Ans. C = 9, Pass 46**

Pink Relay Seat D

The width of the interval is $14\frac{1}{8} - 4\frac{3}{4} = 10 - \frac{5}{8} = 9\frac{3}{8} = \frac{75}{8}$. Between the 3rd and 4th subintervals is $\frac{12}{25}$ of $\frac{75}{8} = \frac{9}{2} = 4\frac{1}{2}$. Add this to $4\frac{3}{4} = 9\frac{1}{4}$. Pass: $\frac{4D+9}{X} = \frac{4\left(\frac{37}{4}\right)+9}{46} = 1$. **D = $9\frac{1}{4}$, Pass 1**

Pink Relay   Seat E

Using $(2^N - 1)(2^{N-1})$ and $2^N - 1$ is a prime number. Plugging in 1 into $2^N - 1$ produces 1 which is not prime. 2 produces 6. 3 = 7(4) = 28. 4 will be prime. 5 produces 31(16) = 496.

6 + 28 + 496 = 530. Pass: $\sqrt{E-X} = \sqrt{530-1}$ = 23. **Ans. E = 530, Pass 23**

Yellow Relay   Seat A

J = .3K or $\frac{J}{K} = \frac{3}{10}$ ➔ $\frac{3J}{2K} = \frac{3\cdot 3}{10\cdot 2} = \frac{9}{20}$ = 45%. Pass: $\frac{3}{5}(45)$ = 27. **Ans. A = 45, Pass 27**

Yellow Relay   Seat B

1000 – 76 = 924. 924/3 = 308 cents per box. 308(16) = 4928. 5000 – 4928 = 72.

Pass: 3X – B = 3(27) – 72 = 9. **Ans. B = 72, Pass 9**

Yellow Relay   Seat C

$\frac{AB}{AC} = \frac{BD}{DC}$ ➔ $\frac{15}{25} = \frac{3}{5} = \frac{24-DC}{DC}$ ➔ 3DC = 120 – 5DC ➔ 8DC = 120, DC = 15.

Pass: $\frac{5X+3C}{9} = \frac{5(9)+3(15)}{9} = \frac{45+45}{9}$ = 10. **Ans. C = 15, Pass 10**

Yellow Relay   Seat D

Width of interval = $16\frac{1}{8} - 4\frac{1}{4} = 12 - \frac{1}{8} = 11\frac{7}{8} = \frac{95}{8}$. Between the 2nd and 3rd subdivision is 9/30 or 3/10 of the distance ➔ $\frac{3}{10}\cdot\frac{95}{8} = \frac{57}{16} = 3\frac{9}{16}$. Add this to $4\frac{1}{4} = 7\frac{13}{16}$.

Pass: $\frac{100X}{D} = \frac{100(10)}{\frac{125}{16}} = \frac{1000(16)}{125}$ = 8(16) = 128. **Ans. D = $7\frac{13}{16}$, Pass 128**

Yellow Relay   Seat E

Using $(2^N - 1)(2^{N-1})$ and $2^N - 1$ is a prime number. We went through the first three in Pink E. We finished with N = 6. For 7: $2^7 - 1 = 127$ is prime. 127(64) = 8128.

Pass: $\sqrt{\frac{E+64}{X}} = \sqrt{\frac{8128+64}{128}} = \sqrt{\frac{8192}{128}} = \sqrt{64}$ = 8. **Ans. E = 8128, Pass 8**

Answer Sheet – States 2018

Individuals – Round 1	Individuals – Round 2	Individuals – Round 3
1. 69	1. 144	1. 650
2. 20	2. 26.4	2. $4\sqrt{5}$
3. 4032	3. 1020º	3. 18

Individuals – Round 4	Individuals – Round 5	Individuals – Round 6
1. 37/32	1. 192	1. – 42
2. 144/5	2. $14\sqrt{2}$	2. ¼
3. 187	3. $0 < k \leq$ ¼	3. 9

Team Round 1

1. 315	4. 11,111,111	7. 19
2. 3	5. 7	8. 84/167
3. – 140	6. 19	

Team Round 2

1. 2/5	4. 25	7. 240
2. 6,999	5. 45º	8. 16
3. 6	6. 64	

	Blue Relay		Green Relay		Pink Relay		Yellow Relay	
	Ans	Pass	Ans	Pass	Ans	Pass	Ans	Pass
Seat A	8	64	11	169	60	36	45	27
Seat B	$\frac{7}{8}$	280	$\frac{10}{13}$	104	193	13	72	9
Seat C	$3\frac{3}{4}$	1050	$4\frac{1}{8}$	429	9	46	15	10
Seat D	9	35	13	33	$9\frac{1}{4}$	1	$7\frac{13}{16}$	128
Seat E	$\frac{\sqrt{21}}{7}$	15	$\frac{\sqrt{21}}{3}$	77	530	23	8128	8