Individuals Round 1 States 2017

3 pts 1. Determine the Greatest Common Factor for 154 and 242.

Ans. __________

4 pts 2. What is the probability of choosing an odd prime number from the set of the first 50 positive integers?

Ans. __________

5 pts 3. Two lines intersect at (4, 5). Each line crosses the line x = 10. One crosses at (10, 10) and the other at (10, -3). Find the sum of the values of the y-coordinates of the y-intercepts of both lines.

Ans. __________

Individuals Round 2 States 2017

3 pts 1. Solve ax + b = cx – 1 for x.

Ans. ______________

4 pts 2. Three equilateral triangles, each with a height of 6, form an isosceles trapezoid. Find the area of the trapezoid. Express in simplest form.

Ans. ___________________

5 pts 3. If the equation of the parabola, having points (6, 0), (2, -4) and (10, 0), takes on the form $y = ax^2 + bx + c$ or $x = ay^2 + by + c$, find a + b + c.

Ans. _________________

Individuals Round 3 States 2017

3 pts 1. The mean of the set of numbers x, 5, -2, x, 7, x, 9 is 4. What is the sum of the mean, median, mode and range?

Ans. _______________

4 pts 2. The volume of a rectangular pyramid is 40. The length and width of the base, and the height form three consecutive integers. Find the sum of these three dimensions.

Ans. _______________

5 pts 3. Find all values of x, such that $\left|x^2-4\right| \geq |4x|$.

Ans. ____

Individuals Round 4 States 2017

3 pts 1. Find all pairs of positive prime numbers whose sum is 24. Express answer in ordered pair form: (smaller, larger)

Ans. ____

4 pts 2. The sum of the measures of angles A, B, C, D is 180º. $\angle$B is the complement of $\angle$C, $m\angle A = 2(m\angle B)$, $m\angle C = 1.25\ (m\angle D)$. Find the $m\angle A + m\angle B$.

Ans. _______________

5 pts 3. Find all value(s) of x such that $\sqrt{9x+12}-\sqrt{3x-2}=\sqrt{6x+2}$.

Ans. ___________________________

Individuals Round 5 States 2017

3 pts 1. Evaluate $\begin{vmatrix} 2 & -3 & 4 \\ 4 & 2 & -3 \\ 3 & 4 & -2 \end{vmatrix}$. **Ans.** _______________

4 pts 2. If $x * y = 2x - y^2$ and $x \# y = x/y$, find (2 * 7) # (7 * 2) in simplest form.

Ans. ____________________

5 pts 3. The ellipse $9x^2 + 4y^2 - 18x + 8y - 23 = 0$ is moved 4 units to the right, then raised 5 units and finally rotated 90° about the center of the ellipse. What are the coordinates of the endpoints of the major axis once it has been moved and rotated?

Ans. __________________________

Individuals Round 6 States 2017

3 pts 1. An item was sold for $25.97 after a 6% sales tax was charged. How much money was paid for the sales tax?

Ans. ________________

4 pts 2. If $\frac{x^{a^2}}{x^{b^2}} = x^{24}$, where x > 1 and a + b = 4. Find the value of a – b.

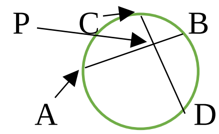
Ans. ________________

5 pts 3. $\log_5 6 + \log_{625} 2 + \log_{25} 3 = \log_5 N$. Find N in simplest radical form.

Ans. ______________________

Team Round 1 States 2017

4 pts 1. In the circle, PB = $\frac{2}{3}$CP. If AP = 4, find the measure of DP.



(1) Ans. _______________ 4 pts

4 pts 2. A rectangular pyramid has base dimensions of 2 yards 1foot 3 inches by 1 yard 2 inches. Its height is 16 inches. Find its volume in cubic feet.

(2) Ans____________________ 4 pts

6 pts 3. The bill for Cable TV, Internet and telephone service is changing. Currently Cable is 60% of the bill. Internet service is 20% of the bill. Telephone service is 20% of the bill. Internet is increasing by 50% over its former price. Telephone service is staying at the same price. Cable is increasing by 25% of its former price. The bill for the first month of changes is $250. What would have been the bill using the former charges?

(3) Ans. ___________________6 pts

6 pts 4. If x + y + 2z = 1, 3x – 4y – 5z = 4½ and 4x + 3y + 2z = 2, find x + y + z.

(4) Ans. _______________6 pts

6 pts 5. Simplify: $\frac{2x+3}{x+5}-\frac{3x+2}{x-5}+\frac{2x^2+13x+55}{x^2-25}$ **(5) Ans. _______________________ 6 pts**

8 pts 6. At what point does the function $f(x)=\frac{4}{x-1}-\frac{x+3}{x^2-1}$ cross one of its asymptotes?

(6) Ans. ______________ 8 pts

8 pts 7. Series A = $a_1, a_2, a_3, \ldots$; Series B = $b_1, b_2, b_3, \ldots$; Series C = $a_1+b_1, a_2+b_2, a_3+b_3, \ldots$.

Series A and B are arithmetic. In Series A: the 21st term is 345 and the 35th term is 527. In Series B: the 17th term is 287 and the 38th term is 602. What is the sum of the first 38 terms of C?

(7) Ans. __________________ 8 pts

8 pts 8. Find all θ where $0^o \le \theta < 360^o$ for which $\cos^2\theta\csc\theta - \cot\theta = \sin\theta$.

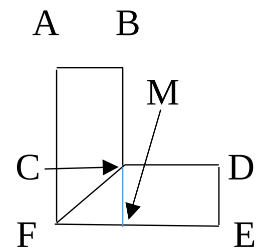
(8) Ans. ____________________________________ 8 pts

Team Round 2 States 2017

4 pts 1. $a^{5/4} \cdot b^{1/6} \cdot c^{3/2} = \sqrt[q]{a^n b^p c^r}$, where n, p, r, q are the smallest possible integers. Find the value of n + p + r + q.

(1) Ans. ____________________ **4 pts**

4 pts 2. Given: $\overline{AB} \parallel \overline{FE} \parallel \overline{CD}$, $\overline{AF} \parallel \overline{BC} \parallel \overline{DE}$, $\overline{BM} \perp \overline{FE}$, BC = CD, DE = 4, M the midpoint of $\overline{FE}$, and FC = 5. Find the area of hexagon ABCDEF.



(2) Ans. ____________________ **4 pts**

6 pts 3. Ben takes 12 hrs. to do a certain job. Al can do the same job in 16 hrs. Al starts the job at 6:00 AM. Ben comes to work at 8:00 AM. Before they finish Al takes a 2 hour lunch break. To the nearest minute at what time will they finish the job? Specify AM or PM.

(3) Ans. ____________________ **6 pts**

6 pts 4. If $\log_{x-2}\left[(x^3-6x^2+12x-8)(\sqrt{x-2})\right]=n$, find *n*. **(4) Ans.** ____________________ **6 pts**

6 pts 5. If $\frac{2+3i}{4-i}(1+i)+(2-i)^{-1}=a+bi$, find a + b in simplest form.

(5) Ans. ____________________ **6 pts**

8 pts 6. Find the domain of *f(x)*, if $f(x)=\dfrac{3}{1-\dfrac{2}{x-\dfrac{1}{x-2}}}$.

(6) Ans. ____ **8 pts**

8 pts 7. Quadrilateral ABCD has coordinates: A(-5, 12), B(-3, -9), C(10, -2) and D(16, 21).

Find the area of the quadrilateral.

(7) Ans. ____________________ **8 pts**

8 pts 8. If x ≠ -3, -1, or 3/5, find all value(s) of x so that $\frac{x+2}{x+3}-\frac{x-2}{x+1}=\frac{2x+1}{5x-3}$.

(8) Ans. ________________________________ **8 pts**

Blue Relay Seat A States 2017

The lines 8x – 5y = -3 and 4x – 3y = -5 intersect at point A. Find the sum of the coordinates of A.

Pass back: 2A A = Your answer.

Blue Relay Seat B States 2017

Car A travels at 80 mph and car B travels at 90 mph on an oval 2 mile track. How many miles does B travel before he gains a lap on A?

Pass back: $\frac{X+B}{2}$ B = Your answer X = The number you receive.

Blue Relay Seat C States 2017

Two opposite angles of a kite measure 60^o and 90^o. If each of the longer pair of sides is 12, its area takes on the form m + p$\sqrt{q}$. Find the value of m + p + q.

Pass back: 1.5X + C C = your answer X = The number you receive.

Blue Relay Seat D States 2017

Find the largest value of x which does not satisfy: $|2x-5|<4x+3$.

Pass back: DX – 2 D = Your answer X = The number you receive

Blue Relay Seat E States 2017

A sock drawer has two pairs of brown socks, 3 pairs of blue socks and 3 pairs of black socks. Two socks are selected at random from the drawer. What is the probability that they are both the same color?

Pass in: $\frac{X}{E}$ E = Your answer X = The number you receive

Green Relay Seat A States 2017

The lines 9x + 7y = -3 and 4x + 5y = 10 intersect at point B. Find the product of the coordinates of B.

Pass back: $-\frac{1}{2}$A A = Your answer

Green Relay Seat B States 2017

Car A travels at 80 mph and car B travels at 90 mph on an oval 2 mile track. How many miles does A travel before B gains a lap on A?

Pass back: 2X – B B = Your answer X = The number you receive.

Green Relay Seat C States 2017

Two opposite angles of a kite measure 60^o and 90^o. If each of the longer pair of sides is 6, its area takes on the form m + p$\sqrt{q}$. Find the value of m + p + q.

Pass back: 2C – X C = Your answer X = The number you receive

Green Relay Seat D States 2017

Find the smallest value of x which does not satisfy $|2x-5| > 4x+3$.

Pass back: X - 7D D = Your answer. X = The number you receive.

Green Relay Seat E States 2017

A sock drawer has two pairs of brown socks, 3 pairs of blue socks and 3 pairs of black socks. Two socks are selected at random from the drawer. What is the probability that they are not the same color?

Pass in: $\frac{X}{E}$ E = Your answer X = The number you receive.

Pink Relay Seat A States 2017

If $\dfrac{15x^3y^2 \cdot 16x^2y^5}{24x^4y^4} = ax^m y^n$, find the value of amn.

Pass back: $\frac{1}{3}$A A = Your answer

Pink Relay Seat B States 2017

Mark has \$3.32. Larry has \$4.48. How much money in dollars should Mark give Larry, so that Mark will then have only one-half as much as Larry?

Pass back: 10BX B = Your answer X = The number you receive.

Pink Relay Seat C States 2017

An 8-15-17 triangle is similar to a smaller triangle whose shortest side is 6. What is the area of the smaller triangle?

Pass back: 4C – X C = Your answer X = The number you receive

Pink Relay Seat D States 2017

If $\begin{bmatrix} 4 & 3 \\ 2 & -1 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$, find the value of a + b + c + d + e + f.

Pass back: X - $\frac{D}{3}$ D = Your answer X = The number you receive.

Pink Relay Seat E States 2017

If $0^\circ \le \theta < 360^\circ$, find the positive difference between the maximum and minimum values of the solutions for the equation $\tan^2\theta + \sin^2\theta = 2 - \cos^2\theta$.

Pass in: X + $\frac{E}{10}$ E = Your answer X = The number you receive

Yellow Relay Seat A States 2017

If $\dfrac{30x^5y^7 \cdot 24x^2y^4}{36x^3y^3} = ax^my^n$, find $\dfrac{am}{n}$.

Pass back: 2A A = Your answer

Yellow Relay Seat B States 2017

Mark has $3.50. Larry has $4.30. How much money in dollars should Mark give Larry so that Mark will have one fifth as much as Larry?

Pass back: BX B = Your answer X = The number you receive.

Yellow Relay Seat C States 2017

A 9-12-21 triangle is similar to a larger triangle whose second largest side is 30. What is the perimeter of the larger triangle?

Pass back: $\dfrac{4C+20}{X}$ C = Your answer X = The number you receive.

Yellow Relay Seat D States 2017

If $\begin{bmatrix} 4 & 3 \\ 2 & -1 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$, find the value of a + b + c – (d + e + f).

Pass back: 2X – D D = Your answer X = The number you receive.

Yellow Relay Seat E States 2017

If $0^o \le \theta < 360^o$, find the positive difference between the maximum and minimum values for the solutions of the equation: $\tan^2\theta + \cos^2\theta = 4 - \sin^2\theta$.

Pass in: X - $\dfrac{E}{10}$ E = Your answer X = The number you receive.

Solutions – Individuals Round 1

1. $154 = 2 \cdot 7 \cdot 11$, $242 = 2 \cdot 11 \cdot 11$. GCF = 2(11) = 22. **Ans. 22**

2. PROBLEM CHANGED: Answer: 7/25 or 0.28 or 28% **Ans. 7/25**

3. (4, 5), (10, 10) ➔ Slope = 5/6, y = 5/6x ➔ 5x – 6y = -10, y-i = 5/3.

   (4, 5), 10, -3) ➔ Slope = -8/6, y = -8/6x ➔ 8x + 6y = 62, y-i = 31/3. Sum = 36/3. **Ans. 12**

Individuals Round 2

1. ax + b = cx – 1 ➔ ax – cx = -b – 1 ➔ (a – c)x = -b – 1 ➔ x = $\frac{-b-1}{a-c} = \frac{b+1}{c-a}$. **Ans.** $\frac{b+1}{c-a}$

2. Upper base is $8\sqrt{3}$ long, lower base is $4\sqrt{3}$ long. Area = 1/2 (6)($12\sqrt{3}$) = $36\sqrt{3}$. **Ans.** $\mathbf{36}\sqrt{3}$

3. For zeroes: y = a(x – 6)(x – 10), for (2, -4): -4 = a(-4)(-8), a = - 1/8, y = -1/8(x – 6)(x – 10)

   y = $-\frac{1}{8}x^2 + 2x - 7\frac{1}{2}$. $-\frac{1}{8} + 2 - 7\frac{1}{2}$ = 2 – 7 = $2 - 7\frac{5}{8}$ = $-5\frac{5}{8}$ **Ans.** $\mathbf{-5\frac{5}{8}}$

Individuals Round 3

1. 3x + 19 = 4(7), 3x = 9, so x = 3. Mean = 4, median = 3, mode = 3, range = 11. **Ans. 21**

2. (x – 1)(x)(x+1)/3 = 40 ➔ $x^3 - x = 120$ ➔ $x^3 - x - 120 = 0$ ➔ (x – 4)(x – 5)(x – 6) = 0.

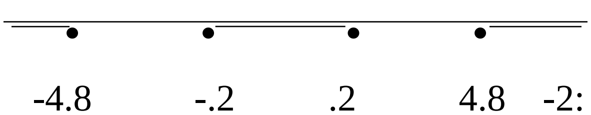
L + W + H = 15. **Ans. 15**

3. Critical points for $|x^2 - 4| \geq |4x|$ are when (1) $x^2 - 4 = 4x$ and (2) $x^2 - 4 = -4x$. In (1):

$x^2 + 4x - 4 = 0$ ➔ x = $\frac{-4 \pm \sqrt{16-4(-4)}}{2} = \frac{-4 \pm \sqrt{32}}{2} = -2 \pm 2\sqrt{2}$. In (2): $x^2 + 4x - 4 = 0$ ➔

$\frac{4 \pm \sqrt{16-4(-4)}}{2} = \frac{4 \pm \sqrt{32}}{2} = 2 \pm 2\sqrt{2}$. Since $\sqrt{2}$ is approximately 1.4, then in (1): $-2 - 2\sqrt{2} \doteq -4.8$,

$-2 + 2\sqrt{2} \doteq -.2$, $2 + 2\sqrt{2} \doteq 4.8$, and $2 - 2\sqrt{2} \doteq .2$. Placing these on a number line:



Plugging in interval points: -5: 21 ≥ 20, yes; -2: 0 ≥ 4, no; 0: 4 ≥ 0, yes; 2: 0 ≥ 8, no; 5: 21 ≥ 20, yes

Ans. $\mathbf{x \leq -2 - 2\sqrt{2}}$ **or** $\mathbf{2 - 2\sqrt{2} \leq x \leq -2 + 2\sqrt{2}}$ **or** $\mathbf{x \geq 2 + 2\sqrt{2}}$

Individuals Round 4

1. Primes less than 24: 2, 3, 5, 7, 11, 13, 17, 19, 23. **Ans. (5, 19), (7, 17), (11, 13)**

2. Let C = x. Then A = 180 – 2x, B = 90 – x, C = x, d = 4/5x ➔ 270 – $\frac{6}{5}$x = 180, 90 = $\frac{6}{5}$x.

x = 75. So A = 180 – 150 = 30, and B = 90 – 75 = 15. A + B = 45. **Ans. 45**

3. $\sqrt{9x+12}-\sqrt{3x-2}=\sqrt{6x+2}$ ➔ 9x + 12 - 2$\sqrt{9x+12}\sqrt{3x-2}$ + 3x – 2 = 6x + 2 ➔

6x + 8 = 2$\sqrt{9x+12}\sqrt{3x-2}$ ➔ 3x + 4 = $\sqrt{27x^2+18x-24}$ ➔ $9x^2 + 24x + 16 = 27x^2 + 18x - 24$

0 = $3x^2$ -6x – 40 ➔ 0 = (3x + 4)(3x – 5), so x = - $\frac{4}{3}$ or $\frac{5}{3}$. - $\frac{4}{3}$ does not work. **Ans. 5/3**

Individuals Round 5

1. $\begin{vmatrix} 2 & -3 & 4 \\ 4 & 2 & -3 \\ 3 & 4 & -2 \end{vmatrix}$ = -8 + 27 + 64 – (24 – 24 + 24) = 83 – 24 = 59. **Ans. 59**

2. x * y = 2x – y^2 and x # y = x/y. 2 * 7 = 2(2) – 7^2 = -45. 7 * 2 = 2(7) – 2^2 = 10.

-45 # 10 = -4510 = -4.5 **Ans. -4.5**

3. 9(x^2 – 2x +1) + 4(y^2 + 2y + 1) = 23 + 13 = 36 or $\frac{(x-1)^2}{4}+\frac{(y+1)^2}{9}=1$ has center at (1, -1). If moved 4 units to the right, the center would be (5, -1). If the raised 5 units, the center would now be at (5, 4). If now rotated 90° then the endpoints of the major axis would now end up on the horizontal axis: (5 ± 3, 4) = (8, 4) and (2, 4). **Ans. (8, 4) and (2, 4)**

Individuals Round 6

1. 1.06x = 25.97, x = 25.97/1.06 = 24.50. Tax = 25.97 – 24.50 = $1.47 **Ans. $1.47**

2. $\frac{x^{a^2}}{x^{b^2}}=x^{24}$ ➔ $x^{a^2-b^2}=x^{24}$, thus $(a-b)(a+b)=24$ ➔ a – b = $\frac{24}{a+b}=\frac{24}{4}$ = 6. **Ans. 6**

3. $\log_5 6+\log_{625} 2+\log_{25} 3=\log_5 N$ ➔ $\log_5 6+\log_5 2^{\frac{1}{4}}+\log_5 3^{\frac{1}{2}}=\log_5 6\cdot 2^{\frac{1}{4}}\cdot 3^{\frac{2}{4}}$ =

$\log_5 6\sqrt[4]{18}$. **Ans. $6\sqrt[4]{18}$**

Team Round 1

1. AP(PB) = CP(DP) ➔ 4(2/3x) = x(DP) ➔ 4(2/3) = DP = 8/3= $2\frac{2}{3}$. **Ans. $2\frac{2}{3}$**

2. Converting each to feet: $\frac{1}{3}\left(\frac{29}{4}\right)\left(\frac{4}{3}\right)\left(\frac{19}{6}\right)=\frac{1}{3}\left(\frac{551}{18}\right)=\frac{1}{3}\left(30\frac{11}{18}\right)=10\frac{11}{54}$ **Ans. $10\frac{11}{54}$**

3. Original bill: Cable + Internet + Telephone: .6x + .2x + .2x,

new bill: 1.25(.2x) + 1.5(.2x) + .2x = 250 ➔ 1.25x = 250, so x = 200. **Ans. 200**

4. (1) x + y + 2z = 1, (2) 3x – 4y – 5y = 4½, (3) 4x + 3y + 2z = 2. –(1) + (3) and 5(1) + 2(2):

(4) 3x + 2y = 1 and (5) 11x – 3y = 14. 3(4) + 2(5): 9x + 6y = 3 and 22x – 6y = 28. Adding these two 31x = 31, so x = 1. In (4): 3(1) + 2y = 1, so y = -1. In (1): (1) + (-1) + 2z = 1,

so z = $\frac{1}{2}$. x + y + z = $\frac{1}{2}$. **Ans. 1/2**

5. $\frac{2x+3}{x+5}-\frac{3x+2}{x-5}+\frac{2x^2+13x+55}{x^2-25}$ ➔ $\frac{(2x+3)(x-5)-(3x-2)(x+5)+2x^2+13x+55}{(x-5)(x+5)}$ ➔

$\frac{2x^2-7x-15-(3x^2+17x+10)+2x^2+13x+55}{(x-5)(x+5)}$ ➔ $\frac{x^2-11x+30}{(x-5)(x+5)}$ ➔ $\frac{\cancel{(x-5)}(x-6)}{\cancel{(x-5)}(x+5)}$ **Ans.** $\frac{x-6}{x+5}$

6. $f(x)=\frac{4}{x-1}-\frac{x+3}{x^2-1}$ = $\frac{4(x+1)}{(x-1)(x+1)}-\frac{x+3}{x^2-1}=\frac{4x+4-x-3}{x^2-1}$ = $\frac{3x+1}{x^2-1}$. A function cannot cross a vertical asymptote, but it can cross a slant or horizontal one. So f(x) = $\frac{3x+1}{x^2-1}$, as x increases without bound, f(x) tends to 0. f(x) = 0 is a horizontal asymptote. Setting these equal to each other tells where they intersect: $\frac{3x+1}{x^2-1}$ = 0. So 3x + 1 = 0, or x = -1/3. The point is (-1/3, 0).

Ans. (-1/3, 0)

7. In Series A: (1) 345 = a + 20d; (2) 527 = a + 34d. (2) – (1): 182 = 14d, so d = 13. In (1): 345 = a + 20(13) ➔ a = 345 – 260 = 85. So Series A = 85 + 13d.

In Series B: (1) 287 = a + 16d and (2) 602 = a + 37d. (2) – (1): 315 = 21d, so d = 15. In (1): 287 = a + 16(15) ➔ a = 287 – 240 = 47. So Series B = 47 + 15d. This makes Series C equal to C = 132 + 28d. The sum of the first 38 terms: 38^{th} term = 132 + 28(37) = 132 + 1036 = 1068;

Sum = $\frac{38(132+1168)}{2}$ = 19(1300) = 24700. **Ans. 24,700**

8. $\cos^2\theta\csc\theta-\cot\theta=\sin\theta$ ➔ $\frac{\cos^2\theta}{\sin\theta}-\frac{\cos\theta}{\sin\theta}=\sin\theta$ ➔ $\cos^2\theta-\cos\theta=\sin^2\theta$ ➔

$\cos^2\theta-\cos\theta=1-\cos^2\theta$ ➔ $2\cos^2\theta-\cos\theta-1=0$ ➔ (2 cos θ + 1)(cos θ – 1) = 0. So cos = 1, which is at 0° which cannot be used since csc 0° is undefined; or cos θ = -1/2, which is at 120° or 240°.

Ans. 120° or 240°

Team Round 2

1. $a^{5/4}\cdot b^{1/6}\cdot c^{3/2}=a^{\frac{15}{12}}\cdot b^{\frac{2}{12}}\cdot c^{\frac{18}{12}}=\sqrt[12]{a^{15}b^2c^{18}}$, 12 + 15 + 2 + 18 = 47. **Ans. 47**

2. FM = 3, since ΔFCM is a 3-4-5Δ. Therefore ME = CD = BC = 3. Thus the area of quadrilateral ABMF = 3(7) = 21, and the area of Quadrilateral CDEM = 12. **Ans. 33**

3. $\frac{1}{16}(2+T-2)+\frac{1}{12}T=1$ ➔ 3T + 4T = 48 ➔ 7T = 48 , so T = $6\frac{6}{7}$ hrs. $\frac{6}{7}(60) = \frac{360}{7}=51\frac{3}{7}$ min

Since Ben started at 8:00 AM, he finished at 2:51 PM to the closest minute. **Ans. 2:51 PM**

4. $\log_{x-2}(x^3-6x^2+12x-8)(\sqrt{x-2})=n$ ➔ $x^3-6x^2+12x-8=(x-2)^3$ **Ans. 3 ½**

5. $\frac{2+3i}{4-i}(1+i)+(2-i)^{-1}$ ➔ $\frac{(2+3i)(1+i)}{4-i}+\frac{1}{2-i}$ ➔ $\frac{-1+5i}{4-i}+\frac{1(2+i)}{(2-i)(2+i)}$ ➔ $\frac{(-1+5i)(4+i)}{(4-i)(4+i)}+\frac{1(2+i)}{5}$

$\frac{-9+19i}{17}+\frac{2+i}{5} = \frac{5(-9+19i)+17(2+i)}{85} = \frac{-45+95i+34+17i}{85} = \frac{-11+112i}{85}$. a + b = $\frac{101}{85}$. **Ans. 101/85**

6. $f(x)=\frac{3}{1-\frac{2}{x-\frac{1}{x-2}}}$, x ≠ 2; $f(x)=\frac{3}{1-\frac{2}{\frac{x^2-2x-1}{x-2}}} = \frac{3}{1-\frac{2x-4}{x^2-2x-1}}$, x = $\frac{2\pm\sqrt{4-4(-1)}}{2}=1\pm\sqrt{2}$

So x ≠ 1 ± $\sqrt{2}$. *f(x)* = $\frac{3}{\frac{x^2-2x-1-(2x-4)}{x^2-2x-1}} = \frac{3}{\frac{x^2-4x+3}{x^2-2x-1}} = \frac{3x^2-6x-3}{x^2-4x+3}$, $x^2-4x+3=0$ ➔

(x – 3)(x - 1) = 0, so x ≠ 1 or 3. **Ans. All Reals except 1, 2, 3 or 1 ± $\sqrt{2}$**

7. Using determinants for the area of ΔABC + area of ΔACD:

$$\frac{1}{2}\left\|\begin{matrix}-5 & 12 & 1\\ -3 & -9 & 1\\ 10 & -2 & 1\end{matrix}\right\| = \frac{1}{2}(45+120+6+90-10+36) = \frac{1}{2}(287) = 143\frac{1}{2}$$

$$\frac{1}{2}\left\|\begin{matrix}-5 & 12 & 1\\ 10 & -2 & 1\\ 16 & 21 & 1\end{matrix}\right\| = \frac{1}{2}(10+192+210+32+105-120) = \frac{1}{2}(429) = 214\frac{1}{2}.$$

Ans. 358

8. $\frac{x+2}{x+3}-\frac{x-2}{x+1}=\frac{2x+1}{5x-3}$ ➔ (x+2)(x+1)(5x–3) – (x–2)(x+3)(5x–3) = (2x+1)(x+3)(x+1) ➔

$(x^2+3x+2)(5x-3) - (x^2+x-6)(5x-3) = (2x+1)(x^2+4x+3)$ ➔ $(2x+8)(5x-3) = 2x^3+9x^2+10x+3$ ➔

$10x^2+34x-24 = 2x^3+9x^2+10x+3$ ➔ $0 = 2x^3 - x^2 - 24x + 27$.

3	2	-1	-24	27
		6	15	-27
	2	5	-9	0

So x = 3 and for $2x^2 + 5x - 9 = 0$: $x = \frac{-5\pm\sqrt{25-4(-18)}}{4} = \frac{-5\pm\sqrt{97}}{4}$.

Ans. 3 or $\frac{-5\pm\sqrt{97}}{4}$

Blue Relay Seat A

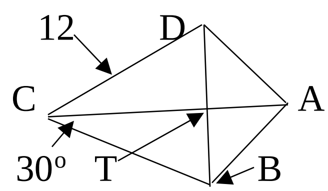
(1) 8x – 5y = -3 and (2)4x – 3y = -5: (1) -2(2): -5y + 6y = -3 + 10, y = 7. In (2): 4x – 3(7) = -5 4x = 16, so x = 4. 4 = 7 = 11. Pass: 2A = 22. **Ans. A = 11, Pass: 22**

Blue Relay Seat B

Car A: 80T = D. Car B: 90T = D + 2. 90T = 80T + 2, 10T = 2, so T = 1/5. 80(1/5) = 16. So car A travelled 16 miles and car B 18 miles. Pass: $\frac{X+B}{2}=\frac{22+18}{2}=20$. **Ans. B = 18, Pass: 20**

Blue Relay Seat C

In the kite at right $\angle DAB$ is a right angle, AD = AB, DC = BC and BC = 12. DB = 12, CT = $6\sqrt{3}$ and AT = 6. Since $\overline{AC}\perp\overline{BD}$ then the area is half the product of the diagonals:



Area $=\frac{1}{2}(12)(6+6\sqrt{3})=36+36\sqrt{3}$. m + p + q = 36 + 36 + 3 = 75.

Pass: 1.5X + C = 1.5(20) + 75 = 30 + 75 = 105 **Ans. C = 75, Pass: 105**

Blue Relay Seat D

$|2x-5|<4x+3$, critical points area where (1) 2x - 5 = 4x + 3 or (2) 2x – 5 = -4x – 3. In (1): -8 = 2x, so x = -4. In (2): 6x = 2, so x = 1/3. Plugging in interval points: -5: 15 < -17, no; 0: 5< 3, no; 1: 3 < 7, yes. So x < 1/3. Largest value not satisfying is 1/3. Pass: DX – 2 = (1/3)(105) – 2 = 35 – 2 = 33.

Ans. D = 1/3, Pass: 33

Blue Relay Seat E

2 pairs brown, 3 pairs blue, 3 pairs black. P(2 same color): 2 brown or 2 blue or 2 black ➔

$\frac{{}_4C_2 + {}_6C_2 + {}_6C_2}{{}_{16}C_2} = \frac{6+15+15}{120} = \frac{36}{120} = \frac{3}{10}$. Pass: $\frac{X}{E} = 33 \div \frac{3}{10} = 110$. **Ans. E = 3/10, Pass: 110**

Green Relay Seat A

(1) 9x + 7y = -3, (2) 4x + 5y =10. 5(1) -7(2): 45x – 28x = -15 – 70 ➔ 17x = -85, x = -5. In (1): 9(-5) + 7y = -3, 7y = 42, y = 6. -5(6) = -30. Pass: - (1/2)(-30) = 15. **Ans. A = -30, Pass: 15**

Green Relay Seat B

Refer to Blue Seat B: 80(1/5) = 16. Pass: 2X – B = 2(15) – 16 = 14. **Ans. B = 16, Pass: 14**

Green Relay Seat C

Refer to Blue Seat C: $\frac{1}{2}(6)(3+3\sqrt{3}) = 9+9\sqrt{3}$. m + p + q = 21. Pass: 2C – X = 2(21) – 14 = 28.

Ans. C = 21, Pass: 28

Green Relay Seat D

The solution to this inequality is x < 1/3. Thus, 1/3 is the smallest value that does NOT satisfy the problem. Pass: X – 7D = 28 – 7(1/3) = 77/3. **Ans. D = 1/3, Pass: 77/3**

Green Relay Seat E

From Blue Seat E, the complement is 7/10. Pass: $\frac{X}{E} = \frac{77}{3} \cdot \frac{10}{7} = \frac{110}{3}$. **Ans. E = 7/10, Pass: 110/3**

Pink Relay Seat A

$\frac{15x^3y^2 \cdot 16x^2y^5}{24x^4y^4} = ax^m y^n = 10xy^3$, amn = 30. Pass: $\frac{1}{3}A = \frac{1}{3}(30) = 10$. **Ans. A = 30, Pass: 10**

Pink Relay Seat B

3.32 – x = ½(4.48 + x) ➔ 6.64 – 2x = 4.48 + x ➔ 2.16 = 3x, so x = .72.

Pass: 10BX = 10(.72)(10) = 72. **Ans. B = .72, Pass: 72**

Pink Relay Seat C

Since the 8-15-17Δ is a right Δ, its area is 60. The ratio of the areas of the two Δ's is $\left(\frac{4}{3}\right)^2 = \frac{16}{9}$.

Thus $\frac{16}{9} = \frac{60}{x}$, 16x = 60(9), x = $\frac{15(9)}{4} = \frac{135}{4} = 33\frac{3}{4}$. Pass: 4C – X: $4\left(\frac{135}{4}\right)$ - 72 = 63.

Ans. C = $33\frac{3}{4}$, Pass: 63

Pink Relay Seat D

$\begin{bmatrix} 4 & 3 \\ 2 & -1 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 32 \\ -7 & 6 \\ 2 & 41 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$. a + b + c + d + e + f = 75. Pass: X - $\frac{D}{3}$ = 63 - $\frac{75}{3}$ = 38.

Ans. D = 75, Pass: 38

Pink Relay Seat E

$\tan^2\theta + \sin^2\theta = 2 - \cos^2\theta$ ➔ $\tan^2\theta + \sin^2\theta + \cos^2\theta = 2$ ➔ $\tan^2\theta + 1 = 2$ ➔ $\tan^2\theta = 1$ ➔

$\tan\theta = \pm 1$, thus $\theta = 45, 135, 225, 315$. $315 - 45 = 270$. Pass: X + $\frac{E}{10}$ = 38 + $\frac{270}{10}$ = 65.

Ans. E = 270, Pass: 65

Yellow Relay Seat A

$\frac{30x^5y^7 \cdot 24x^2y^4}{36x^3y^3} = 20x^4y^8 = ax^my^n$, $\frac{am}{n} = \frac{20 \cdot 4}{8}$ = 10. Pass: 2A = 20. **Ans. A = 10, Pass: 20**

Yellow Relay Seat B

3.50 – x = (1/5)(4.30 + x) ➔ 17.50 – 5x = 4.30 + x ➔ 13.20 = 6x, so x = 2.20.

Pass: BX = (2.20)(20) = 44. **Ans. B = 2.20, Pass: 44**

Yellow Relay Seat C

The 9-12-21Δ has a perimeter of 42. So $\frac{12}{42} = \frac{30}{x}$, 12x = 30(42) ➔ x = 5(21) = 105.

Pass: $\frac{4C + 20}{X} = \frac{4(105) + 20}{44} = \frac{420 + 20}{44}$ = 10. **Ans. C = 105, Pass: 10**

Yellow Relay Seat D

From Pink Seat D: 1 + 32 – 7 – (6 + 2 + 41) = 26 – 49 = -23. Pass: 2X – D = 2(10) – (-23) = 20 + 23 = 43. **Ans. D = -23, Pass: 43**

Yellow Relay Seat E

$\tan^2\theta + \cos^2\theta = 4 - \sin^2\theta$, as in Pink Seat E, $\tan^2\theta = 3$ ➔ $\tan\theta = \pm\sqrt{3}$, therefore

θ = 60, 120, 240, 300. 300 – 60 = 240. Pass: X - $\frac{E}{10}$ = 43 - $\frac{240}{10}$ = 19. **Ans. E = 240, Pass: 19**

Answer Sheet – States 2017

Individuals Round 1

1. **22**
2. **7/25**
3. **12**

Individuals Round 2

1. $\frac{b+1}{c-a}$
2. **36**$\sqrt{3}$
3. **-5**$\frac{5}{8}$ or **-45/8**

Individuals Round 3

1. **21**
2. **15**
3. $x \le -2 - 2\sqrt{2}$ **or** $2 - 2\sqrt{2} \le x \le -2 + 2\sqrt{2}$ **or** $x \ge 2 + 2\sqrt{2}$

Individuals Round 4

1. **(5, 19), (7, 17), (11, 13)**
2. **45** or **45º**
3. **5/3**

Individuals Round 5

1. **59**
2. **-4 ½** or **-9/2**
3. **(8, 4) and (2, 4)**

Individuals Round 6

1. **1.47** or **$1.47**
2. **6**
3. $6\sqrt[4]{18}$

Team Round 1

1. **2**$\frac{2}{3}$ or **8/3**
2. **10**$\frac{11}{54}$ or **551/54**
3. **200** or **$200**
4. **1/2**
5. $\frac{x-6}{x+5}$
6. **(-1/3, 0)**
7. **24,700**
8. **120, 240** or **120º, 240º**

Team Round 2

1. **47**
2. **33**
3. **2:51 PM**
4. **3½** or **7/2** or **3.5**
5. **101/85**
6. **All Reals ≠ 1 ±** $\sqrt{2}$ **, 1, 2 or 3**
7. **358**
8. $\frac{-5\pm\sqrt{97}}{4}$ **or 3**

Relays

Blue	Ans.	Pass	Green	Ans.	Pass	Pink	Ans.	Pass	Yellow	Ans.	Pass
A	11	22	A	-30	15	A	30	10	A	10	20
B	18	20	B	16	14	B	.72	72	B	2.20	44
C	75	105	C	21	28	C	33$\frac{3}{4}$	63	C	105	10
D	1/3	33	D	1/3	77/3	D	75	38	D	-23	43
E	3/10	110	E	7/10	110/3	E	270	65	E	240	19