1 Individuals States 2015

3 pts 1. For real numbers a and b, $|b| = 5$ and $a^2 = 36$. What is the least possible value of $a - b$?

Ans. ________________

4 pts 2. When n is an integer greater than 2, the function L(n) outputs the measure (in degrees) of one of the interior angles of a regular n-gon. The function G(n) outputs the number of diagonals in a convex n-gon. If $(L \cdot G)$(n) = 1800, give the specific name for the polygon that satisfies these conditions.

Ans. ____________________

5 pts 3. Let k be the second smallest integer greater than one that is a perfect square, a perfect cube and a perfect fourth power. How many positive integral divisors does k have?

Ans. ________________

2 Individuals States 2015

3 pts 1. Two sides of a scalene triangle have lengths of 7 and 4. What is the sum of the possible integral lengths of the third side?

Ans. ________________

4 pts 2. Find the quotient when $1 - x^2$ is divided by $1 - x^{-2}$.

Ans. ____________________

5 pts 3. If $\log_3(20x + 9) - \log_3(10x - 9) = \log_3(5x - 4)$, find x.

Ans. ________________

3 Individuals States 2015

3 pts 1. The points (3, 4), (5, -6) and (11, m) are collinear. Find m.

Ans. ________________

4 pts 2. For the system: $\begin{aligned} 2x - y + z &= 12 \\ 3x + 6y - z &= 2 \\ x + y + 6z &= 1 \end{aligned}$ find the value of x + y + z.

Ans. ________________

5 pts 3. Evaluate: $$\begin{vmatrix} \sin\frac{3\pi}{2} & \sin(2\pi) & \cos(4\pi) \\ \left(\sin\frac{\pi}{3}\right)^2 & \sec 0 & \cot\left(\frac{3\pi}{4}\right) \\ \cos\left(\frac{9\pi}{2}\right) & \tan\left(\frac{-5\pi}{4}\right) & \left(\csc\left(\frac{\pi}{4}\right)\right)^4 \end{vmatrix}$$

Ans. ________________

4 Individuals States 2015

3 pts 1. If $3^3 4^5 6^4 = 12^P$, find P.

Ans. ________________

4 pts 2. Pete has a box without a lid. The box has dimensions 12 units by 13 units by 5 units. He wants to paint the box, inside and out. Each can of paint covers 10 units^2 of surface. He can only buy whole cans of paint. How many cans of paint will Pete need to purchase to paint the box, if the lid would have covered the side with the largest area?

Ans. ________________

5 pts 3. Three vertices are chosen randomly from the vertices of a cube. What is the probability that the three points chosen will make a non-right triangle?

Ans. ________________

5 Individuals States 2015

3 pts 1. The vertices of a 9 inch by 12 inch rectangular piece of paper are labeled ABCD with AB = 12 inches. The paper is folded along a line segment PQ that is 2 inches from $\overline{AB}$ and parallel to $\overline{AB}$. After being folded, point A is how many inches closer to point C?

Ans. ________________

4 pts 2. In the lawless land of Mathmania, math symbols mean different things. In Mathmania Arithmetic, a – b means to multiply a and b, a + b means to divide a by b, a x b means to subtract a from b, and a $\div$b means add a to the square of b. Evaluate the following Mathmania Arithmetic expression:

((((5 – 4) $\div$ 5) + 3) x 2) **Ans.** _________________

5 pts 3. The Whispering Chamber at the local science museum is the shape of an ellipse. If two students stand at the foci with their backs to each other, even a slight whisper from one student will be reflected to the other student's ear. The students are located 8 meters apart, each facing a wall one meter away. Find the length of the minor axis of the ellipse.

Ans. _________________

6 Individuals States 2015

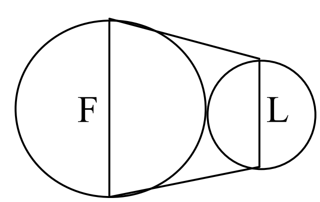
3 pts 1. Simplify: $\dfrac{(2-5i)(3+7i)+(5+3i)(2+7i)}{3+4i}$, where $i = \sqrt{-1}$.

Ans. ______________________

4 pts 2. Find X, if $X > 0$ and $X \bullet X \bullet X \bullet X = X + X + X + X$.

Ans. ______________________

5 pts 3. Two circles are externally tangent to each other such that the diameter of Circle F is one base of an isosceles trapezoid and the diameter of Circle L is the other base, as shown in the figure. If the circumference of F is twice the circumference of L and the distance between their centers is $\frac{1}{\pi}$, what is the area of the trapezoid in term of π?



Ans. ______________________

Team Round 1 States 2015

4 pts 1. Three distinct points A, B and C lie on a circle. Chords AC and BC have the same length and minor arc AC has a measure equal to three times the measure of $\angle$ACB. Find the measure of major arc ABC.

(1) Ans. _______________ 4 pts

4 pts 2. Joanne's new salary is 40% higher than her old salary. She now gives weekly 10% of her new salary to charity. If the amount she now gives to charity is \$168, what was her old weekly salary?

(2) Ans. ________________4 pts

6 pts 3. A triangle has interior angles of 45^o, 60^o and 75^o. If the side opposite the 60^o angle has a length of $4\sqrt{6}$, find the area of the triangle.

(3) Ans. _____________________ 6 pts

6 pts 4. Find all values of x such that $6x^4 + 17x^3 - 36x^2 - 57x + 70 = 0$.

(4) Ans. ___________________________ 6 pts

6 pts 5. A convex decagon has interior angles with integer degree values. At least nine of the interior angles have the same measure. The last angle may or may not have the same measure. What is the smallest possible degree measure of each of the nine congruent angles?

(5) Ans. ________________6 pts

8 pts 6. A sequence defined as $a_1 = 7, a_2 = 8, a_3 = 9$ and for n > 3, $a_n = a_{n-1} - a_{n-2} + a_{n-3}$. Find the value of a_{2015}.

(6) Ans. ________________8 pts

8 pts 7. Suppose f(x) is a real-valued function, such that $5f\left(\frac{1}{x}\right) + \frac{f(2x)}{x^2} = x$ for $x \neq 0$.

Find f(1).

(7) Ans. ________________8 pts

8 pts 8. Find all values of x such that: $\frac{2}{2x+3} \le \frac{5}{5-x}$ and $6x^2 + 5x - 50 \le 0$.

(8) Ans. ____ 8 pts

Team Round 2 States 2015

4 pts 1. Given that $5^2 \bullet 3^8 = x^y$, where both x and y are positive integers, find the smallest possible value for x + y.

(1) Ans. ________________ 4 pts

4 pts 2. Five couples at a prom stand in a line to have a group picture taken by a photographer. How many distinguishable arrangements for the picture are possible, if each couple stays together?

(2) Ans. ________________ 4 pts

6 pts 3. Find all values of x, such that

$$\frac{4x+1}{x+1} - \frac{6x-7}{x+3} = \frac{x-2}{5x+5}$$

(3) Ans ___________________________ 6 pts

6 pts 4. Express $\sqrt{10!}$ in simplest form as $a!\sqrt{b}$.

(4) Ans. _____________________ 6 pts

6 pts 5. If 0 < x < 1, and the value of $\frac{2015}{x}$ is an integer, what is the least possible value of this integer?

(5) Ans. _____________________ 6 pts

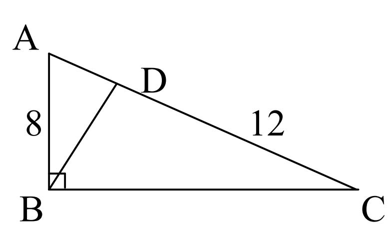
8 pts 6. The terms $1, \sin\theta, \cos^2\theta, \frac{-\sqrt{2}}{4}$ form a geometric sequence, in that order, Give the value of $\sec^2\theta$ - $|\tan\theta|$.

(6) Ans. ________________ 8 pts

8 pts 7. The hour hand and minute hand of a certain clock are each 6 inches long. Over time, the outer tips of the hands sweep out a circle. Find the exact area of the smaller sector formed by the hands at 8:21.

(7) Ans. ________________ 8 pts

8 pts 8. In right triangle ABC, $\overline{BD}$ is perpendicular to $\overline{AC}$. AB = 8 and DC = 12. Find the perimeter of ΔABC.



(8) Ans. ____________________________ 8 pts

Blue Relay – Seat A States 2015

Find the value of x for the system: $\frac{9}{x}+\frac{3}{y}=0$ and $\frac{3}{x}+\frac{5}{y}=3$

Pass back: -A A = Your answer

Blue Relay – Seat B States 2015

Nola has 83 candy bars. Pete has 34. How many candy bars should Nola give to Pete, so that Pete will then have four-fifths as many as Nola?

Pass back: XB B = Your answer X = The number you will receive

Blue Relay – Seat C States 2015

Given a triangle with sides lengths of 9, 40 and 41, what is the area of the triangle?

Pass back: 3X – C C = Your answer X = The number you will receive

Blue Relay – Seat D States 2015

If $7^k = 289$, what is the value of $7^{\frac{k}{2}+1}$?

Pass back: D – 2X D = Your answer X = The number you will receive

Blue Relay – Seat E States 2015

If p and q are roots of the equation $x^2 - 20x + 10$, then $p^2q + pq^2 =$

Pass in: $(E – 4X)^2$ E = Your answer X = The number you will receive

Green Relay – Seat A   States 2015

Find y, if $\frac{5}{x}+\frac{4}{y}=-1$ and $\frac{2}{x}-\frac{1}{y}=10$.

Pass back: $\frac{-2}{A}$ A = Your answer

Green Relay – Seat B   States 2015

Peter has 70 candy bars. Bob has 49. How many bars should Bob give Peter so that Peter will have two and a half times as many as Bob?

Pass back: BX   B = Your answer   X = The number you will receive

Green Relay – Seat C   States 2015

Given that a triangle with side lengths of 20, 21 and 29, what is the area of the triangle?

Pass back: $\frac{C}{X-15}$ C = Your answer   X = The number you will receive

Green Relay – Seat D   States 2015

If 6^{k} = 256, what is the value of $6^{\frac{k}{2}+1}$?

Pass back: $\frac{3XD}{4}$ D = Your answer   X = The number you will receive

Green Relay – Seat E   States 2015

If p and q are roots of the equation x^2 - 16x + 8 = 0, then $p^2q+pq^2=$

Pass in: $E\sqrt{X}$ E = Your answer   X = The number you will receive

Pink Relay – Seat A   States 2015

The operation *x, y* is defined as *x, y* = $x^3 - y^3$. If the two roots of $x^2 + 5x + 6 = 0$ are a and b where a < b, find *a, b*.

Pass back: A – 1   A = Your answer

Pink Relay – Seat B   States 2015

A jar has $7 in it, composed of nickels, dimes, quarters and half-dollars. There are twice as many dimes as quarters, five times as many nickels as quarters and three more half-dollars than dimes. How many quarters are in the jar?

Pass back: $\frac{-X}{B}$ B = Your answer   X = The number you will receive

Pink Relay – Seat C   States 2015

The exterior angle of a regular polygon measures 45^o. How many diagonals does the polygon have? There is only one exterior angle at each vertex.

Pass back: XC   C = Your answer   X = The number you will receive

Pink Relay – Seat D   States 2015

What is the constant term in the expansion of $\left(x - \frac{2}{x^2}\right)^9$?

Pass back: $\frac{D+32}{-X}$ D = Your answer   X = The number you will receive

Pink Relay – Seat E   States 2015

Find 2A – B given that $\frac{A}{x-3} + \frac{B}{x+7} = \frac{3x+41}{x^2+4x-21}$

Pass in: $(E - X)^3$ E = Your answer   X = The number you will receive

Yellow Relay – Seat A States 2015

The operation *x, y* is defined as *x, y* = $x^3 - y^3$. If the roots of $x^2 + 7x + 12 = 0$ are *a* and *b* with $a < b$, find *a**, *b**.

Pass back: -A A = Your answer

Yellow Relay – Seat B States 2015

A jar has $7 in it, composed of nickels, dimes, quarters and half-dollars. There are twice as many dimes as quarters, five times as many nickels as quarters and 3 more half-dollars than dimes. How many half-dollars are in the jar?

Pass back: $\frac{X+5}{B}$ B = Your answer X = The number you will receive

Yellow Relay – Seat C States 2015

The exterior angle of a regular polygon measures 36^o. How many diagonals does the polygon have? There is only one exterior angle at each vertex.

Pass back: $\frac{X^3}{C+1}$ C = Your answer X = The number you will receive

Yellow Relay – Seat D States 2015

What is the constant term in the expansion of $\left(x^2 - \frac{2}{x}\right)^6$?

Pass back: $\frac{D}{X+6}$ D = Your answer X = The number you will receive

Yellow Relay – Seat E States 2015

Find 3A + 2B given that $\frac{A}{x+3} + \frac{B}{x-3} = \frac{5x+3}{x^2-9}$

Pass in: $(X + E)^2$ E = Your answer X = The number you will receive

Solutions – Individuals – Round 1

1. a = ± 6 and b = ± 5 Smallest value of a – b = (-6) – (5) = -11. **Ans. -11**

2. $G(n) \bullet L(n) = \frac{n(n-3)}{2} \bullet \frac{180(n-2)}{n}$ = 90(n – 2)(n – 3) = 1800 ➔ $n^2 - 5n + 6 = 20$ ➔ $n^2 - 5n - 14 = 0$ ➔ (n – 7)(n + 2) = 0. So n = 7. **Ans. Septagon**

3. LCM(2, 3, 4) = 12. The second smallest # would b 3^{12}. It has 13 divisors. **Ans. 13**

Individuals – Round 2

1. The 3rd side must have sides from 4 to 10. $\cancel{4}$, 5, 6, $\cancel{7}$, 8, 9, 10 . Sum = 38. **Ans. 38**

2. $\frac{1-x^2}{1-\frac{1}{x^2}} = \frac{1-x^2}{\frac{x^2-1}{x^2}} = (1 - x^2)\left(\frac{x^2}{-(1-x^2)}\right) = -x^2$. **Ans. $-x^2$**

3. $\log_3(20x + 9) - \log_3(10x - 9) = \log_3(5x - 4)$ ➔ $\frac{20x+9}{10x-9} = 5x - 4$ ➔
$20x + 9 = (10x - 9)(5x - 4) = 50x^2 - 85x + 36$ ➔ $0 = 50x^2 - 105x + 27$ =
(5x – 9)(10x + 3). So x = 9/5 or -3/10. -3/10 does not work. **Ans. 9/5**

Individuals – Round 3

1. Using slope: $\frac{4-(-6)}{3-5} = \frac{m-(-6)}{11-5}$ ➔ $\frac{10}{-2} = \frac{m+6}{6}$ ➔ -30 = m + 6. m = -36. **Ans. -36**

2. Adding both side yields: 6x + 6y + 6z = 15, so x + y + z = 2½ . **Ans. 2½**

3. $$\begin{vmatrix} \sin\frac{3\pi}{2} & \sin(2\pi) & \cos(4\pi) \\ \left(\sin\frac{\pi}{3}\right)^2 & \sec 0 & \cot\left(\frac{3\pi}{4}\right) \\ \cos\left(\frac{9\pi}{2}\right) & \tan\left(\frac{-5\pi}{4}\right) & \left(\csc\left(\frac{\pi}{4}\right)\right)^4 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 1 \\ \frac{3}{4} & 1 & -1 \\ 0 & -1 & 4 \end{vmatrix} = \left(-4 - \frac{3}{4}\right) + (1) = -3\tfrac{3}{4}$$ **Ans. -3¾**

Individuals – Round 4

1. $3^3 4^5 6^4 = 3^3 4^5 2^4 3^4 = 3^7 4^5 2^2 = 3^7 4^7 = 12^7$. So p = 7. **Ans. p = 7**

2. 2(12)(13) + 4(13)(5) + 4(5)(12) = 44(13) + 240 = 572 + 240 = $\frac{812}{10}$ = 81+ **Ans. 82**

3. There are 8 vertices, so ${}_8C_3$ = 56 Δ's. From each vertex at the top two equilateral triangles are formed from the diagonals of the faces. 8/56 = 1/7. **Ans. 1/7**

Individuals – Round 5

1. Folding the paper move A so that the original 9-12-15 right triangle is now a 5-12-13 right triangle. 15 side is now 13, 2 inches closer. **Ans. 2 in.**

2. ((((5 – 4) $\div$ 5) + 3) x 2) ➔ 5 – 4 => 5(4) = 20; 20$\div$5 => 20 + 5^2 = 45; 45 + 3 => 45 $\div$ 3 = 15; 15 x 2 => 2 – 15 = -13. **Ans. –13**

3. The length of the major axis 2a is 10, so a = 5. The distance from the center to the foci is 4, so c = 4. In an ellipse $a^2 - b^2 = c^2$, so $25 - b^2 = 16$ ➔ $9 = b^2$, so b = 3. **Ans. 6**

Individuals – Round 6

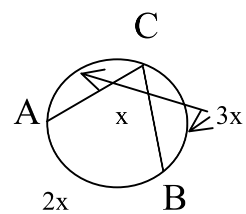
1. $\dfrac{(2-5i)(3+7i)+(5+3i)(2+7i)}{3+4i} = \dfrac{6-i+35+10+41i-21}{3+4i} = \dfrac{30+40i}{3+4i} = \dfrac{10(3+4i)}{3+4i}$. **Ans. 10**

2. $x^4 = 4x$ ➔ $x^4 - 4x = 0$ ➔ $x(x^3 - 4) = 0$. $x = \sqrt[3]{4}$. **Ans. $\sqrt[3]{4}$**

3. The ratio of the radii is equal to the ratio of the circumferences. So let x be the radius of the small circle, so the radius of the larger circle is 2x. The height of the trapezoid is 3x, which is actually $\frac{1}{\pi}$. The area is $\frac{1}{2}$h(B +b) = $\frac{1}{2}$(3x)(4x + 2x) =

$\frac{1}{2}(3x)(2)(3x) = (3x)^2 = \frac{1}{\pi^2}$. **Ans. $\frac{1}{\pi^2}$**

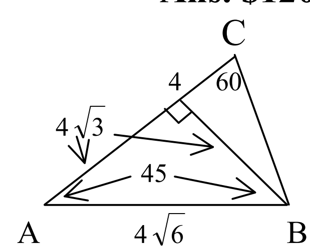
Team – Round 1



1. At right let m$\angle$ACB = x. Then measures of minor arcs AC, AB, and BC are 3x, 2x, and 3x respectively. 8x = 360, so x = 45^o, and major arc ABC = 5x = 225^o.

Ans. 225^o

2. If Joanne gives 10% to charity, which is \$168, then her new salary is \$1680. 1.40S = 1680, so 1680/1.4 = 1200. **Ans. \$1200**



3. In the figure at right, draw altitude from B to meet $\overline{AC}$ at D. ΔBDC is a 30-60-90Δ. Since ΔABD is a 45-45-90Δ, then AD = $4\sqrt{3}$ and so does BD. DC is thus 4. The area of the triangle is $\frac{1}{2}(4+4\sqrt{3})4\sqrt{3} = 8\sqrt{3} + 24$.

Ans. 24 +8$\sqrt{3}$

4. By synthetic division:

	6	17	-36	-57	70
1		6	23	-13	-70
	6	23	-13	-70	
-2		-12	-22	70	
	6	11	-35		

1 and -2 are two of the solutions. Solving $6x^2 + 11x - 35 = 0$ will produce the other two: (3x – 5)(2x + 7) = 0 So x = 5/3 and -7/2.

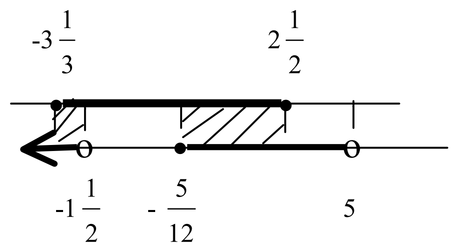
Ans. 1, -2, 5/3, -7/2

5. The sum of the interior angles is 8(180) = 1440. The largest possible angle for the 10th angle is 180, which it cannot be, but if you subtract this from 1440, then the smallest of the 9 congruent angles must be greater than (1440 – 180)/9 = 1260/9 = 140. **Ans. 141^o**

6. $a_n = a_{n-1} - a_{n-2} + a_{n-3}$ ➔ $a_4 = 9 - 8 + 7 = 8$; $a_5 = 8 - 9 + 8 = 7$; $a_6 = 7 - 8 + 9 = 8$; $a_7 = 8 - 7 + 8 = 9$; $a_8 = 9 - 8 + 7 = 8$. Sequence: 7, 8, 9, 8, 7, 8, 9, 8, … . So the pattern repeats 7, 8, 9, 8 , every 4 terms. Thus 2015/4 gives a remainder of 3. **Ans. 9**

7. $5f\left(\frac{1}{x}\right) + \frac{f(2x)}{x^2} = x$. In the left fraction, if x = 1, then (1): 5f(1) + f(2) = 1. In the right fraction, if x = 1/2 , then (2): 5f(2) + 4f(1) = 1/2. Multiply (1) by -5: -25f(1) – 5f(2) = -5. Adding this to (2): -21f(1) = -4½. f(1) = (-4½)/(-21) $= \frac{9}{2} \bullet \frac{1}{21} = \frac{3}{14}$. **Ans. 3/14**

8. $\frac{2}{2x+3} \le \frac{5}{5-x}$ ➔ Critical Points are -1½, 5 and where $\frac{2}{2x+3} = \frac{5}{5-x}$, thus 10 – 2x = 10x + 15 -5 = 12x, so x = -5/12. For $6x^2 + 5x - 50 \le 0$, critical points are where $6x^2 + 5x - 50 = 0$ ➔ (2x – 5)(3x + 10) = 0, so x = 2½ or -3⅓.



Using the number lines at right: for algebraic fraction is graphed on the bottom and the polynomial on top.

Plugging in interval points for the bottom: -2: -2 ≤ 5/7, yes

-1: 2 ≤ 5/6, no 0: 2/3 ≤ 1, yes and 6: 2/15 ≤ -5, no.
Plugging in to (2x-5)(3x+1) ≤ 0 for the top: -4: $-\bullet- \le 0$, no; 0 $-\bullet+ \le 0$, yes; 3: $+\bullet- \le 0$, no. **Ans. -3⅓ ≤ x < -1½ or -5/12 ≤ x ≤ 2½**

Team – Round 2

1. $5^2 \bullet 3^8 = x^y$ ➔ $5^2 \bullet (3^4)^2 = (5 \cdot 81)^2 = 405^2 = x^y$. 405 + 2 = 407. **Ans. 407**

2. The 5 couples are ${}_5P_5$. Each couple makes 2 arrangements. 5!2!2!2!2!2! **Ans. 3840**

3. $\frac{4x+1}{x+1} - \frac{6x-7}{x+3} = \frac{x-2}{5x+5}$ ➔ 5(4x+1)(x+3) – 5(6x-7)(x+1) = (x-2)(x+3) ➔ $20x^2 + 65x + 15 - 30x^2 + 5x + 35 = x^2 - x - 6$ ➔ $0 = 11x^2 - 69x - 56 = (11x + 8)(x - 7)$
Ans. 7 or -8/11

4. $\sqrt{10!} = \sqrt{10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2} = \sqrt{(5\cdot2)(3\cdot3)(4\cdot2)\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2} = \sqrt{5(2\cdot3)(3\cdot4\cdot2)7\cdot6\cdot5\cdot4\cdot3\cdot2} = 6\bullet5\bullet4\bullet3\bullet2\sqrt{7} = 6!\sqrt{7}$. **Ans. $6!\sqrt{7}$**

5. If x = $\frac{2015}{2016}$, then 2015 divided by x is 2015 times $\frac{2016}{2015}$ = 2016. **Ans. 2016**

6. If the first term is 1 and the second term is $\sin\theta$, then the common ratio is $\sin\theta$. The third term is $\sin\theta$ times $\sin\theta$, which should be $\sin^2\theta$, but it's $\cos^2\theta$. That means that these are equal, that could only happen at 45^o or any multiple there of. Since the fourth term is $-\sqrt{2}/4$, then θ must be a second or fourth quarter angle. In either case $\sec^2\theta = 2$ and $|\tan\theta| = 1$. Therefore $\sec^2\theta - |\tan\theta| = 2 - 1 = 1$. **Ans. 1**

7. Each minute on the clock is $\frac{360}{60} = 6^o$. From 20 minutes after to 20 minutes of is 120^o. 21 after means this would make it 114^o. The amount of degree change by the hour hand from the 8 to 9 is $\frac{21}{60}$ times the 5 minutes (30^o) = $\frac{21}{60} \bullet 30 = 10½^o$. $114^o + 10½^o = 124½^o$.

Area = $\frac{124½}{360}\pi 36 = \frac{124½}{10}\pi = \frac{249}{20}\pi$. **Ans.** $\frac{249}{20}\pi$

8. Let AD = x. Then $\frac{x}{8} = \frac{8}{x+12}$ ➔ $x^2 + 12x = 64$ ➔ $x^2 + 12x - 64 = 0$ ➔

(x + 16)(x - 4) = 0. AD = 4. That makes ΔADC and the other triangles 30-60-90Δ's.
So BC = $8\sqrt{3}$, and the perimeter of ΔABC = 24 + $8\sqrt{3}$. **Ans. 24 + 8$\sqrt{3}$**

Blue Relay – Seat A

Let u = 1/x and v = 1/y, then (1) 9u + 3v = 0 and (2) 3u + 5v = 3. -1/3(1) = -3u – v = 0.
Adding this to (2): 4v = 3 or v = 3/4. In (1): 9u + 3(3/4)= 0 ➔ 9u = -9/4, so u = -1/4.
Thus x = -4. Pass back: -(-4) = 4. **Ans. A = -4, Pass back: 4**

Blue Relay – Seat B

4/5(83 – x) = 34 + x ➔ 4(83 – x) = 5(34 + x) ➔ 332 – 4x = 170 + 5x ➔ 162 = 9x
x = 18. Pass back: XB = 4(18) = 72. **Ans. B = 18, Pass back: 72**

Blue Relay – Seat C

The 9-40-41Δ s a right triangle. Area = $\frac{1}{2}$(9)(40) = 180. Pass: 3(72) – 180 = 36

Ans. C = 180, Pass back: 36

Blue Relay – Seat D

4. $7^{\frac{k}{2}+1} = \left(7^k\right)^{1/2} \bullet 7^1 = (289)^{1/2}(7) = 17(7) = 119$. Pass back: 119 – 2(36) = 47

Ans. D = 119, Pass back: 47

Blue Relay – Seat E

5. $p^2q + pq^2$ = pq(p + q). In the equation $x^2 - 20x + 10 = 0$, pq = 10 and p + q = 20.
So pq(p + q) = 10(20) = 200. Pass in: $(200 - 4(47))^2 = 12^2 = 144$.

Ans. E = 200, Pass in: 144

Green Relay – Seat A

As in Blue Seat A: (1) 5u + 4v = -1 and (2) 2u – v = 10. -2(1) + 5(2): -8v – 5v = 2 + 50,
-13v = 52, so v = -4 and y = -1/4. Pass back: $\frac{-2}{-1/4} = 8$. **Ans. A = – ¼, Pass back: 8**

Green Relay – Seat B

70 + x = 2½(49 - x) ➔ 2(70 + x) = 5(49 – x) ➔ 140 + 2x = 245 – 5x ➔
7x = 105, so x = 15. Pass back: 15(8) = 120. **Ans. B = 15, Pass back: 120**

Green Relay – Seat C

Like Blue C: area = ½(20)(21) = 210. Pass back: $\frac{210}{120-15} = 2$ **Ans. C = 210, Pass back: 2**

Green Relay – Seat D

$6^{\frac{k}{2}+1} = (6^k)^{\frac{1}{2}} \bullet 6^1 = (256)^{\frac{1}{2}} \bullet 6 = 16 \bullet 6 = 96$. Pass back: $\frac{3(2)(96)}{4} = 144$.

Ans. D = 96, Pass back: 144

Green Relay – Seat E

As in Blue E for $x^2 - 16x + 8 = 0$: 16(8) = 128. Pass in: $128\sqrt{144} = 128(12) = 1536$

Ans. E = 128, Pass in: 1536

Pink Relay – Seat A

$x^2 + 5x + 6 = 0$ ➔ $(x + 3)(x + 2) = 0$. So solutions are -2, -3. $(-3)^3 - (-2)^3 = -27 + 8 = -19$
Pass back: -19 – 1 = -20. **Ans. A = -19, Pass back:-20**

Pink Relay – Seat B

5(5x) + 10(2x) + 25x + 50(2x – 3) = 700 => 25x + 20x + 25x + 100x – 150 = 700 ➔
170x = 850, then x = 5. 5 quarters. Pass back: $\frac{-(-20)}{5} = 4$. **Ans. B = 5, Pass back: 4**

Pink Relay – Seat C

$\frac{360}{45} = 8$. It has 8 sides. $\frac{8(8-3)}{2} = 20$. It has 20 diagonals. Pass back: 4(20) = 80.

Ans. C = 20, Pass back: 80

Pink Relay – Seat D

$\binom{9}{6}x^6\left(\frac{-2}{x^2}\right)^3 = 84(-8) = -672$. Pass back: $\frac{(-672)+32}{-80} = \frac{-640}{-80} = 8$.

Ans. D = -672, Pass back: 8

Pink Relay – Seat E

A(x + 7) + B(x – 3) = 3x + 41. If x = 3, then 10A = 50, so A = 5. If x = -7, then -10B = 20
So B = -2. 2A – B = 2(5) – (-2) = 12. Pass back: $(12 - 8)^3 = 64$.

Ans. E = 12, Pass back: 64

Yellow Relay – Seat A

The roots of $x^2 + 7x + 12$ are -3 and -4. *-4, -3* = $(-4)^3 - (-3)^3 = -64 + 27 = -37$.
Pass back: -(-37) = 37. **Ans. A = -37, Pass back: 37**

Yellow Relay – Seat B

From Pink Seat B, there are (2x - 3) or (2(5) – 3) = 7 half-dollars. Pass back: $\frac{37+5}{7} = 6$

Ans. B = 7, Pass back: 6

Yellow Relay – Seat C

$\frac{360}{36} = 10$, there are 10 sides and thus $\frac{10(10-3)}{2} = 35$ diagonals. Pass back: $\frac{6^3}{35+1} = 6$.

Ans. C = 35, Pass back: 6

Yellow Relay – Seat D

$\binom{6}{4}(x^2)^2\left(\frac{-2}{x}\right)^4 = 15(-2)^4 = 15(16) = 240$. Pass back: $\frac{240}{6+6} = 20$

Ans. D = 240, Pass back: 20

Yellow Relay – Seat E

A(x – 3) + B(x + 3) = 5x + 3. If x = -3, then -6A = -12, so A = 2. If x = 3, then 6B = 18,
so B = 3. 3A + 2B = 3(2) + 2(3) = 12.Pass in: $(20 + 12)^2 = 32^2 = 1024$.

Ans. E = 12, Pass back: 1024

Answer Sheet – States 2015

Individuals Round 1
1. **-11**
2. **Septagon** or **Heptagon**
3. **13**

Individuals Round 2
1. **38**
2. $-x^2$
3. **9/5** or **1 4/5**

Individuals Round 3
1. **-36**
2. **2½** or **2.5**
3. **-3¾** or **-3.75**

Individuals Round 4
1. **7** or **p = 7**
2. **82** or **82 cans**
3. **1/7**

Individuals Round 5
1. **2** or **2 in.**
2. **-13**
3. **6** or **6 meters**

Individuals Round 6
1. **10**
2. $\sqrt[3]{4}$
3. $\frac{1}{\pi^2}$

Team Round 1
1. 225^o
2. **1200** or **$1200**
3. $24 + 8\sqrt{3}$
4. $1, -2, 1\frac{2}{3}, -3\frac{1}{2}$
5. 141^o
6. **9**
7. **3/14**
8. $-3\frac{1}{3} \le x < -1\frac{1}{2}$ or $-5/12 \le x \le 2\frac{1}{2}$

Team Round 2
1. **407**
2. **3840**
3. **7 or -8/11**
4. $6!\sqrt{7}$
5. **2016**
6. **1**
7. $\frac{249\pi}{20}$ or $12\frac{9}{20}\pi$ or 12.45π
8. $24 + 8\sqrt{3}$

	Blue Relay		Green Relay		Pink Relay		Yellow Relay	
	Answer	Pass	Answer	Pass	Answer	Pass	Answer	Pass
A	-4	4	-1/4	8	-19	-20	-37	37
B	18	72	15	120	5	4	7	6
C	180	36	210	2	20	80	35	6
D	119	47	96	144	-672	8	240	20
E	200	144	128	1536	12	64	12	1024