1 Individuals States 2014

3 pts 1. Two similar rectangles have corresponding sides in the ratio of 5:4. By what percent is the area of the smaller rectangle less than the area of the larger rectangle?

Ans. _________________

4 pts 2. Find the least possible value of positive integer *A* so that the sum of one quadrillion and *A* is a multiple of 99.

Ans. _________________

5 pts 3. Line segment L has endpoints at (0, 0) and (4, 3). Line segment M has endpoints at (0, 0) and (5, 12). Find the cosine of the acute angle formed by L and M at the origin.

Ans. ______________________

2 Individuals States 2014

3 pts 1. From the set of whole numbers {80, 81, 82, … , 89}, find the one that has the most whole number factors.

Ans. _________________

4 pts 2. Pencils are packed in boxes with X pencils in each row and Y rows so that $X \geq Y$, $X < 4Y$, and XY is the number of pencils in the box. In how many ways can 720 pencils be packed into a single box?

Ans. _________________

5 pts 3. After running a 4-lap race on a circular track, Elmer walked 200 feet on a straight line from one point on the track to where he had dropped has hat on another part of the track. The furthest he was from the track on one side of his walk was 20 feet. How long was the race in feet?

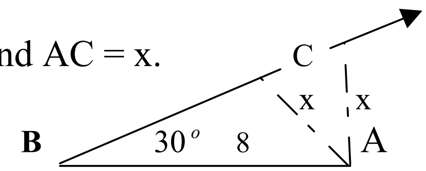
Ans. _________________

3 Individuals States 2014

3 pts 1. A property management company recently announced the renovation of a 122,500 square foot property into a retail store. If the property is only one story tall and is in the shape of a perfect square, how many feet does it measure on a side?

Ans. ________________________

4 pts 2. In triangle ABC, $\angle ABC$ measures 30^o, AB = 8, and AC = x. Find the range of possible values of x.



Ans. ____________________________

5 pts 3. At the Maine State Math Meet, 16 identical souvenir pencils are divided among 10 different players on a team so that each player receives at least 1 pencil. How many different outcomes are possible? Any two outcomes are different if and only if at least one particular player receives a different number of pencils.

Ans. ________________________

4 Individuals States 2014

3 pts 1. Find k so that $\left| \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} -2 & k \\ 1 & -3 \end{bmatrix} \right| = 200$. The upright bars are for determinant, not absolute value.

Ans. ________________________

4 pts 2. How many of the following equal 2.5 when rounded to one decimal place?

2.445	2.45	2.545	$2.\overline{4}$	$2.\overline{45}$
$2.4\overline{5}$	$2.\overline{54}$	$2\frac{9}{20}$	$2\frac{11}{20}$	$\frac{3}{1\frac{11}{49}}$

Ans. _____________

5 pts 3. The floors in the Acme Building where Jenny works are ten feet apart. When Jenny looks out her 1^{st} floor window toward the top of the Math Tower 1200 feet above her height, she looks 60^o up from horizontal. When Cheri looks out her window in the Acme Building, the top of the Math Tower is 30^o up from horizontal. On what floor is Cheri?

Ans. ________________

5 Individuals States 2014

3 pts 1. Calculate: (-19) + (-17) + (-15) + . . . + 55 + 57 + 59. **Ans.** ____________

4 pts 2. If telephone numbers are 7 digits long with a first digit greater than or equal to 1, how many *ascending* telephone numbers are possible? Define an *ascending* telephone number to be one in which each digit after the first is greater than the preceding digit.

Ans. _________________

5 pts 3. Let: $S_1 = \{x \mid x \text{ is an integer such that } 1 \le x \le 20\}$

$S_2 = \{2x \mid x \text{ is an integer such that } 1 \le x \le 20\}$

$S_3 = \{3x \mid x \text{ is an integer such that } 1 \le x \le 20\}$

$S_4 = \{25, 50\}$

Then $S_1 \text{ Y } S_2 \text{ Y } S_3 \text{ Y } S_4$ is the set of every possible score on the throw in a game of darts. Find the number of positive integers less than or equal to 60 that cannot be a score on one throw in a game of darts.

Ans. ________________

6 Individuals States 2014

3 pts 1. How many of the square roots of integers from 1 through 1000 are irrational?

Ans. ________________

3 pts 2. Circle ***O*** is centered at (10, 16). Circle P is externally tangent to circle ***O***. If the equation of Circle P is $x^2 + y^2 - 4x - 2y = 4$, find the greatest value of the y-coordinate of any point on circle ***O***.

Ans. ________________

3 pts 3. A circular racing track surrounds exactly four acres of a grass lawn. Using the facts that there are 640 acres in a square mile and 5280 feet in a mile, determine the circumference of the inside of the track in feet.

Ans. _____________________

1 Team States 2014

4 pts 1. The Law of #16 Rubber Bands is $F = \frac{10}{23}(x - 7)$, where F is the tension force in Newtons, x is the number of centimeters of separation, where $7 \le x \le 38$. Find x in centimeters if F = 12 Newtons.

(1) **Ans.** ________________ **4 pts**

4 pts 2. If the operators (the o's) in the expression 1o2o2 are each replaced with any one of the four operators from the set $\{+, -, \times, \div\}$ and the expression is evaluated, how many different values are possible? (Examples: 1+2-2, 1+2+2)

(2) **Ans.** ________________ **4 pts**

6 pts 3. When the equation $2|x - 1| + |y| = 5$ is graphed, the result is a familiar geometric figure. What is the most restrictive name of this figure?

(3) **Ans.** ______________________ **6 pts**

6 pts 4. A data set contains ninety-five 86's, seventy-six 84's, fifty-seven 82's, thirty-eight 80's, and nineteen 78's. What is the value of the single element that could be added to the data set to make its arithmetic mean equal to exactly 85?

(4) **Ans.** ____________ **6 pts**

6 pts 5. Suppose k has a value such that x + 3 is a factor of $3x^4 + kx^3 - 2x^2 - 8x + 21$. Find the value of the coefficient of x in the quotient resulting from division of the fourth-degree expression by x + 3.

(5) **Ans.** _________________ **6 pts**

8 pts 6. It takes 18 minutes to fill a wading pool using a hose with diameter ½ inch. Assuming the water flows at the same speed, how many minutes would it take to fill the same wading pool using a hose with diameter ¾ inch?

(6) **Ans.** _________________**8 pts**

8 pts 7. Find the sum, $Q_1 + Q_2 + Q_3 + Q_4$, of the next term in each of the following sequences: 97, 101, 103, 107, 109, 113, 115, Q_1 343, 512, 729, 1000, 1313, Q_2 1, 2, 3, 4, 5, 8, 7, Q_3 1, 1, 1, 2, 2, 4, 3, 4, 9, 4, 8, Q_4

(7) **Ans.** _______________**8 pts**

8 pts 8. A game has 7 squares, numbered 0, 1, 2, 3, 4, 5, and 6, on which tokens can be placed. When a token is on square N, for N = 1, 2, 3, 4, 5, a single die is rolled. If the number rolled is greater than N, the token is moved to square N + 1. If the number rolled is less than or equal to N, the token is moved to square N – 1. If the token reaches square 6, the game ends and the token wins. If the token reaches square 0, the game ends and the token loses. Find the probability a token beginning the game on square 1 will ultimately win.

(8) **Ans.** _________________**8 pts**

2 Team   States 2014

4 pts 1. How many integers are NOT in the domain of y = $\frac{\sqrt{x^2-8}}{x+8}$? (1) __________**4 pts**

4 ps 2. Find the greatest prime factor of 2413 = $13^3 + 6^3$? (2) _________ **4 pts**

6 pts 3. Jimmy attempts to count from 1 to 85. As he does so, Joy adds all the numbers Jimmy says. If Jimmy missed one number and Joy's total is 3602, what number did Jimmy skip?

(3) _________ **6 pts**

6 pts 4. According to Kepler's Law, the orbital period is directly proportional to the 2/3 power of the orbital radius, $T = kR^{2/3}$. Moon A of Planet X has an orbital radius of 27 units and an orbital period of 12 days. Find the orbital period in days of Planet X's Moon B, which has an orbital radius of 216 units.

(4) ______________ **6 pts**

6 pts 5. If *a* and *b* can be any integers, how many of the natural numbers in the set {1, 2, 3, 4, … , 99, 100} can be expressed in the form 8*a* + 5*b*?

(5) ______________ **6 pts**

8 pts 6. How many of the following nine cannot be evaluated as finite real numbers?

$\sqrt{2^7 + 20^2 - 23^2}$ $\sin^{-1}(\tan 134^\circ)$ $1 + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \ldots$ $2^{-\infty}$

$\sqrt[5]{-243}$ $|A^{-1}|$, if $A = \begin{bmatrix} -16 & 5 \\ -32 & 10 \end{bmatrix}$ $\dfrac{\begin{vmatrix} 5 & 2 \\ 10 & 8 \end{vmatrix}}{\begin{vmatrix} 12 & 6 \\ 8 & 4 \end{vmatrix}}$ $\log\left[\left(-\frac{1}{16}\right)^5\right]$ $\sec^{-1}\left(\frac{3}{\pi}\right)$

(6) ______________ **8 pts**

8 pts 7. The position of an object moving in the xy-coordinate plane as a function of time (t) is given by the equations: $x(t) = 2\cos t$ and $y(t) = 2\sin t$. If the distance from the object to a point P in the xy-plane is given by the expression $\sqrt{5 + 4\sin t}$, find the coordinates of P.

(7) ____________________ **8 pts**

8 pts 8. A 20 foot length of string of negligible thickness is wrapped around a 10 foot section of cylindrical pipe in a spiral pattern, always maintaining the same angle. If the string circles the pipe ten times between the two ends, what is the diameter of the pipe in feet?

(8) ____________________ **8 pts**

Blue Relay – Seat A

Find the smallest whole number x so that 8758 + x is a multiple of 11.

Pass back: A^2 A = Your answer.

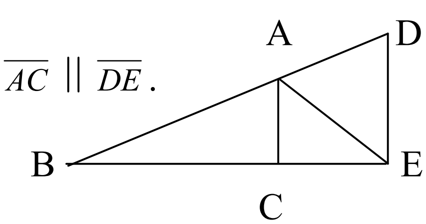
Blue Relay – Seat B

Fence posts of a certain type must be placed no further than 10 feet apart. Find the minimum number of fence posts required to fully enclose a quadrilateral-shaped plot of land measuring 101 feet, 89 feet, 61 feet and 51 feet on its four sides.

Pass back: X – B B = Your answer X = TNYWR

Blue Relay – Seat C

In triangle DBE, AB = 17, BC = 15, AC = 8, DE = 16, and $\overline{AC} \parallel \overline{DE}$. Find AE.



Pass back: $\sqrt{X+C-1}$ C = Your Answer X = TNYWR

Blue Relay – Seat D

Find the remainder if $2x^3 - 15x^2 + 9x - 12$ is divided by x – 7.

Pass back: $\frac{7X+12}{D^2}$ D = Your answer X = TNYWR

Blue Relay – Seat E

Two cards are drawn without replacement from a standard deck of cards and then a fair coin is flipped. Find the probability both cards are clubs and the coin flip is TAILS.

Pass in: $\frac{X}{E}$ E = Your answer X = TNYWR

Green Relay – Seat A

Find the smallest whole number x so that 4764 + x is a multiple of 11.

Pass back: A^2 A = Your answer X = TNYWR

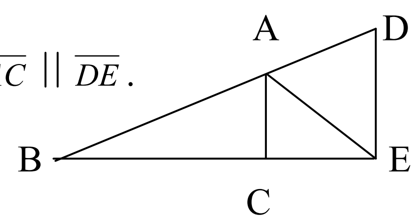
Green Relay – Seat B

Fence posts of a certain type must be placed no further than 10 feet apart. Find the minimum number of fence posts required to fully enclose a quadrilateral-shaped plot of land measuring 91 feet, 82 feet, 51 feet and 49 feet on its four sides.

Pass back: X – B B = Your answer X = TNYWR

Green Relay – Seat C

In triangle DBE, AB = 25, BC = 24, AC = 7, DE = 14 and $\overline{AC} \parallel \overline{DE}$.
Find AE.



Pass back: $\sqrt{X + C + 5}$ C = Your Answer X = TNYWR

Green Relay – Seat D

Find the remainder if $2x^3 - 15x^2 + 9x - 12$ is divided by $x - 6$.

Pass back: $\frac{D-4}{X}$ D = Your answer X = TNYWR

Green Relay – Seat E

Two cards are drawn without replacement from a standard deck of cards and then a fair coin is flipped. Find the probability both cards are jacks and the coin flip is HEADS.

Pass in: $\frac{E^{-1} - 1}{x}$ E = Your answer X = TNYWR

Pink Relay – Seat A

Three posts in a yard at a birthday party must each be labeled with a unique letter selected from the set {A, B, C, D, E, F} for a game. In how many ways can this be done?

Pass back: A – 50 A = Your answer

Pink Relay – Seat B

It requires $7\frac{2}{5}$ bags of manure to fertilize a rectangular garden plot measuring 37 feet by 20 feet. Find the side length in feet of the largest square plot that could be fertilized with 1 bag of manure.

Pass back: $\frac{X}{B}$ B = Your answer X = TNYWR

Pink Relay – Seat C

In right triangle ABC, with AB = 3, BC = 4 and AC = 5, the altitude and the median are both drawn from vertex B to the hypotenuse. How far apart are their intersection points with the hypotenuse? Give answer as a decimal number.

Pass back: $\frac{X}{C}$ C = Your answer X = TNYWR

Pink Relay – Seat D

If the quadratic equation $y = 45x^2 + bx + 20$, where b is a real number, has only one solution and that solution is positive, find it.

Pass back: $D^{-1}X$ D = Your answer X = TNYWR

Pink Relay – Seat E

In right triangle ABC, with AB = 8, BC = 15, and AC = 17, the altitude and median are both drawn from vertex B, intersecting with the hypotenuse at points D and E, respectively. Find the perimeter of triangle DBE. Give answer as a mixed number.

Pass in: EX + 2E E = Your answer X = TNYWR

Yellow Relay – Seat A

Two posts in a yard at a birthday party must each be labeled with a unique letter selected from the set {A, B, C, D, E, F, G} for a game. In how many ways can this be done?

Pass back: $\frac{A}{2}$ A = Your answer

Yellow Relay – Seat B

It requires $1\frac{5}{16}$ bags of manure to fertilize a rectangular garden plot measuring 35 feet by 15 feet. Find the side length in feet of the largest square plot that could be fertilized with 1 bag of manure.

Pass back: BX B = Your answer X = TNYWR

Yellow Relay – Seat C

In right triangle ABC, with AB = 6, BC = 8, and AC = 10, the altitude and the median are both drawn from vertex B to the hypotenuse. How far apart are their intersection points with the hypotenuse? Give answer as a decimal number.

Pass back: $\frac{X}{C}$ C = Your answer X = TNYWR

Yellow Relay – Seat D

If the quadratic equation $y = 25x^2 + bx + 16$, where b is a real number, has only one solution and that solution is positive, find it.

Pass back: $D^{-1}X$ D = Your answer X = TNYWR

Yellow Relay – Seat E

In right triangle ABC, with AB = 9, BC = 12, and AC = 15, the altitude and median are both drawn from vertex B, intersecting with the hypotenuse at points D and E, respectively. Find the perimeter of triangle DBE. Give answer as a mixed number.

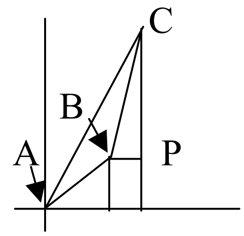
Pass in: EX E = Your answer X = TNYWR

Solutions – Individuals 1

1. Area ratio is 25:16. 9/25 = .36 = 46% **Ans. 36%**

2. Dividing 1,000,000,000,000,000 by 99 leaves remainder of 1 for every 2 zeroes. Since there is an odd number of zeroes, the last number to divide into is 10 thus leaving a remainder of 10. The least value to add is 89. **Ans. 89**

3. Dropping the perpendicular from the point C(5, 13) to the x-axis, and then drawing a horizontal segment from B(4, 3) to meet this segment at point P, as shown makes right triangle PBC with legs of 1 and 9. Thus BC = $\sqrt{82}$. Using cosine law: $(\sqrt{82})^2 = 5^2 + 13^2 - 2(65)\cos CAB$.



$\cos CAB = \dfrac{82-25-169}{-2(65)} = \dfrac{-112}{-2(65)} = \dfrac{56}{65}$. **Ans. 56/65**

Individuals 2

1. 80, 81, 84, and 88 are the ones to have the most number of factors. 80 has 10, 81 has 4, 84 has 12, and 88 has 8. **Ans. 84**

2. Expressing 720 as a product in all possible ways: 1(720), 2(360), 3(240), 4(180), 5(144), 6(120), 8(90), 9(80), 10(72), 12(60), 15(48), 16(45), 18(40), 20(36), 24(30). Assigning these to Y and X, only the last 5 fit the requirements. **Ans. 5**

3. The furthest he was from the track would be at the midpoint of the 200 foot walk. The perpendicular bisect of the 200 foot chord is the diameter of the circular track. By the power of a point theorem, 100(100) = 20x. x = 500. So the diameter of the track is 520π and the length of the race (4 laps) was $4(520\pi) = 2080\pi$. **Ans. 2080π**

Individuals 3

1. $\sqrt{122500} = \sqrt{100\cdot 25\cdot 49} = 10\cdot 5\cdot 7 = 350$. **Ans. 350**

2. The minimum value occurs when $\overline{AC} \perp \overline{BC}$, in which case ΔABC is a 30-60-90Δ and x = 4. By moving C further from B, all values greater than 4 are obtained. **Ans. x $\geq$ 4**

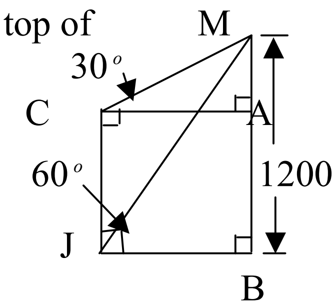
3. Arrange the 16 P's on a line. 9 dividers (bars) can then be placed in the 15 spaces between the P's, no more than one to a space. Person A then gets the pencils to the left of the leftmost bar. Person B gets the pencils to the right of that bar and to the left of the next bar and so on. There are ${}_{15}C_9 = \dfrac{15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10}{6\cdot 5\cdot 4\cdot 3\cdot 2} = 5\cdot 7\cdot 13\cdot 11 = 65\cdot 77$. **Ans. 5005**

Individuals 4

1. $\left|\begin{bmatrix}4 & -1\\ 2 & -3\end{bmatrix}\begin{bmatrix}-2 & k\\ 1 & -3\end{bmatrix}\right| = \begin{vmatrix}-9 & 4k+3\\ -7 & 2k+9\end{vmatrix}$ = - 18k – 81 – (-28k – 21) = 10k – 60 = 200.
10k = 260, so k = 26. **Ans. 26**

2. 2.445 and $2.\overline{4}$ round to 2.4. $2\frac{11}{20}$ = 2.55 round to 2.6. The rest round to 2.5. **Ans. 7**

3 . On the diagram, J = Jenny (first floor), C = Cheri and M = the top of the Math Tower.



Using the 1 - $\sqrt{3}$ - 2 ratio of the sides in a 30-60-90 triangle,

JB = $\frac{1200}{\sqrt{3}} = 400\sqrt{3}$ = AC. Then AM = 400, AB = 800, and

Cheri is 80 floors above Jenny. 1 + 80 = 81.

Ans. 81

Individuals 5

1. 59 = -19 + (n – 1)2 ➔ 78 = (n – 1)2, 39 = n – 1. So there are 40 terms.

Sum = $\frac{40}{2}(-19+59)$ = 20(40) = 800. **Ans. 800**

2. Consider the set {1, 2, 3, 4, 5, 6, 7, 8, 9}. Each ascending telephone number must correspond to a selection of 7 of these 9 numbers, in order. There are therefore:

$_9C_7 = \frac{9\cdot 8}{2}$ = 36 possibilities. **Ans. 36**

3. The union of the four sets includes: all integers ≤ 20; all integers > 20 and ≤ 40 that are factors of 2 and 3, plus 25; all integers > 40 and ≤ 60 that are factors of 3, plus 50. This excludes 23, 29, 31, 35, 37, 41, 43, 44, 46, 47, 49, 52, 55, 56, 58, and 59. 17 #'s. **Ans. 17**

Individuals 6

1. Only square root of perfect squares are rational. 31^2 = 961, 32^2 =1024. Therefore 1000 – 31 = 969. **Ans. 969**

2. $x^2 + y^2 - 4x - 2y = 4$ ➔ $(x-2)^2 + (y-1) = 9$. This circle is centered at (2, 1) and has a radius of 3. Since (2, 1) and (10, 16) are 17 units apart, circle O has a radius of 14. The greatest value of a y-coordinate at any point on O is 16 + 14 = 30. **Ans. 30**

3. Let r = inside radius in feet. Area = πr^2 = 4 acres ➔ $\pi r^2 = \frac{4(5280)^2}{640} = \frac{4\cdot 60^2\cdot 88^2}{10\cdot 64} =$

$4\cdot 6\cdot 60\cdot 11^2 = 2^5\cdot 3^2\cdot 5\cdot 11^2$. Thus r = $\frac{132\sqrt{10\pi}}{\pi}$ and C = $2\pi r = 264\sqrt{10\pi}$ **Ans. 264 $\sqrt{10\pi}$**

Team 1

1. $12 = \frac{10}{23}(x-7)$ ➔ $x - 7 = \frac{276}{10}$ = 34.6. **Ans. 34.6**

2. There are 16 possibilities:

		second operator:			
		+	–	×	÷
first operator:	+	5	1	5	2
	–	1	-3	-3	0
	x	4	0	4	1
	÷	$\frac{5}{2}$	$-\frac{3}{2}$	1	$\frac{1}{4}$

They yield 9 distinct results.

Ans. 9

3. The graph is an equilateral quadrilateral – a parallelogram – with vertices at (-3/2, 0), (1, 5), (7/2, 0), and (1, -5). The most restrictive a rhombus. **Ans. rhombus**

4. Let x be the number to be added. Since the mean must equal 85, the sum of the differences between each element and 85 must be zero.
95(+1) + 76(-1) + 57(-3) + 38(-5) + 19(-7) + 1(x – 85) = 0 ➔
95 – 76 – 171 – 190 – 133 – 85 + x = 0. Thus x = 560. **Ans. 560**

5. -3

3	k	-2	-8	21	
	-9	-3k + 27	9k – 75	-27 + 249	
3	k – 9	-3k + 25	9k – 83	-27 + 270	

So k = 10 and -3(10) + 25 = -5

Ans. -5

6. Since the water speeds are the same, the volume rates will be in the ratio of the Cross-section areas. Small: $\pi\left(\frac{1}{4}\right)^2 = \frac{\pi}{16}$ Large: $\pi\left(\frac{3}{8}\right)^2 = \frac{9\pi}{64}$. The volume rate is 4/9, And the time required is $\frac{4}{9}(18)$ = 8 minutes. **Ans. 8 min.**

7. The first sequence alternates + 4, + 2, + 4, + 2, Q_1 = 119. The second sequence is cubes of consecutive integers, the next is 12^3 = 1728 = Q_2. The third sequence is composed of an arithmetic sequence and a geometric sequence, the next term is 16 = Q_3. The fourth sequence is composed of 3 sequences: the first is an arithmetic sequence, the second is a geometric sequence, and the third is a sequence of squares of consecutive integers, the next is 16 = Q_4. 119 + 1728 + 16 + 16 = 1879. **Ans. 1879**

8.

N:	1	2	3	4	5
move to n + 1:	5/6	4/6	3/6	2/6	1/6
move to n – 1:	1/6	2/6	3/6	4/6	5/6

By symmetry in the probability chart above, a token reaching square 3 has a 1/2 probability of winning or losing. In the first two moves: 1=>2=>3 = 20/36, 1=>2=>1 =

10/36, 1=>0 (on the first move) 6/36. The probability of reaching square 3 without losing is therefore: $\frac{20}{36}+\left(\frac{10}{36}\right)\frac{20}{36}+\left(\frac{10}{36}\right)^2\frac{20}{36}+\cdots=\frac{20/36}{1-10/36}=\frac{20}{26}=\frac{10}{13}$. The probability of ultimately winning is then $\frac{1}{2}\cdot\frac{10}{13}=\frac{5}{13}$. **Ans. 5/13**

Team 2

1. -8 cannot be in the domain. Any integer between $\sqrt{8}$ and -$\sqrt{8}$ cannot be used. Thus -2, -1, 0, 1, 2 cannot be used. There are six. **Ans. 6**

2. Since 2413 can be expressed as a sum of cubes, then the sum of the cube roots must be a factor. Dividing 2413 by 19 produces 127 (which cannot be factored). **Ans. 127**

3. The sum of n integers = $\frac{n(n+1)}{2}$. So $\frac{85(86)}{2}$ = 3655. 3655 – 3602 = 53. **Ans. 53**

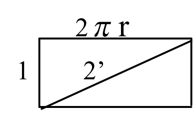
4. For Moon A: 12 = $k(27)^{2/3}=9k$ and k = 4/3. For moon B: T = $\frac{4}{3}(36)$ = 48. **Ans. 48**

5. Since 8 and 5 are relatively prime, every integer can be expressed in the form 8a + 5b an infinite number of ways. So the answer is 100. To find one of the (a, b) ordered pairs for integer n, find b such that n – 5b is a multiple of 8. a = $\frac{n-5b}{8}$. **Ans. 100**

6. (1) $\sqrt{-1}$, so it is **not**. (2) tan 134^o = - tan 46^o, this is less than -1. -1 ≤ $\sin^{-1}$ ≤ 1, so this is **not**. (3) This sequence sums to infinity, so **not**. (4) $2^{-\infty}=\frac{1}{2^{\infty}}$ = 0. (5) $\sqrt[5]{-243}$ = -3. (6) $|A|$ = 0, so A^{-1} is undefined, so it is **not**. (7) The denominator of the denominator is zero, so it is **not**. (8) Cannot take the log of a negative number, so it is **not**. (9) $\frac{3}{\pi}<1$, sec is suppose to be greater than 1, it is **not**. 7 **not**'s. **Ans. 7**

7. Let (a, b) be the coordinates of P. The distance from the object to P is then $\sqrt{(a-2\cos t)^2+(b-2\sin t)^2}=\sqrt{(a^2+b^2+4)-(4a\cos t-4b\sin t}$ this equals $\sqrt{5+4\sin t}$. Thus a = 0, b^2 + 4 = 5, and b = -1. The point is therefore is (0, -1). **Ans. (0, -1)**

8. Imagine that the pipe is covered in a thin film. If the film covering 1 foot of pipe were unwrapped, it would form a rectangle with width of 1 foot and height being the circumference of the pipe. Refer to figure. The diagonal would be the path of the 2 feet of string. By the Pythagorean Theorem: $1^2+4\pi^2r^2=4$ ➔ $r^2=\frac{3}{4\pi^2}$ ➔ r = $\frac{\sqrt{3}}{2\pi}$. Diameter = $\frac{\sqrt{3}}{\pi}$. **Ans.** $\frac{\sqrt{3}}{\pi}$



Blue Relay – Seat A

8758 ÷ 11 has a remainder of 2, so x = 9. Pass $A^2=9^2$ =81. **Ans. 9, Pass 81**

Blue Relay – Seat B

If a side has length L, the number of posts required is $1 + \left[\frac{L}{10}\right]$. The number of posts then is: 101 – 12, 89 – 10 61 – 8, 51 – 7. The total is 37, but the four corner posts have each been counted twice. 37 – 4 = 33. Pass: X – B = 81 – 33 = 48. **Ans. 33, Pass 48**

Blue Relay – Seat C

The side measures in triangle ABC are 8-15-17, so it is a right triangle and $\angle$ACE is 90^o. Since triangles DBE and ABC are similar, CE = 15 and triangle ACE is also an 8-15-17 right triangle with AE = 17. Pass: $\sqrt{X+C-1} = \sqrt{48+17-1} = 8$. **Ans. 17, Pass 8**

Blue Relay – Seat D

By the remainder theorem, the answer is: 2(343) – 15(49) + 9(7) – 12 = 686 – 735 + 63 – 12 = 2. Pass: $\frac{7X+12}{D^2} = \frac{7(8)+12}{2^2} = \frac{68}{4} = 17$. **Ans. 2, Pass 17**

Blue Relay – Seat E

Sequentially, $\frac{1}{4}\cdot\frac{12}{51}\cdot\frac{1}{2} = \frac{1}{34}$. Or, by combinations, $\frac{\binom{13}{2}}{\binom{52}{2}}\cdot\frac{1}{2} = \frac{78}{26\cdot 51\cdot 2} = \frac{1}{34}$.

Pass: $\frac{X}{E} = \frac{17}{\frac{1}{34}} = 17\left(\frac{34}{1}\right) = 578$. **Ans. 1/34, Pass 578**

Green Relay – Seat A

4764 $\div$ 11 has a remainder of 1, so x = 10. Pass: $A^2 = (10)^2$. **Ans. 10, Pass 100**

Green Relay – Seat B

As in Blue Relay B, 91 – 11, 82 – 10, 51 – 7, 49 – 6. Total is 34. 34 – 4 = 30.
Pass: X – B = 100 – 30 = 70. **Ans. 30, Pass 70**

Green Relay – Seat C

Similar to Blue Relay C, AE = 25. Pass: $\sqrt{X+C+5} = \sqrt{70+25+5} = 10$. **Ans. 25, Pass 10**

Green Relay – Seat D

Using synthetic division:

$$\begin{array}{r|rrrr} & 2 & -15 & 9 & -12 \\ 6 & & 12 & -18 & -54 \\ \hline & 2 & -3 & -9 & -66 \end{array}$$

Pass: (D – 4)/X = (-66 – 4)/10 = -7

Ans. -66, Pass -7

Green Relay – Seat E

Sequentially: $\frac{1}{13}\cdot\frac{1}{17}\cdot\frac{1}{2} = \frac{1}{442}$. Pass $\frac{E^{-1}-1}{X} = \frac{442-1}{-7} = -63$. **Ans. 1/442, Pass -63**

Pink Relay – Seat A

There are 6 choices for the first post. 5 remaining for the second post, and 4 remaining for the third. 6(5)4 = 120. Pass: A – 50 = 120 – 50 = 70. **Ans. 120, Pass 70**

Pink Relay – Seat B

37(20) = 740 square feet. $\frac{740}{7.4}$ = 100 square feet per bag. This is a square with each side of length 10 feet. Pass: $\frac{X}{B} = \frac{70}{10} = 7$. **Ans. 10, Pass 7**

Pink Relay – Seat C

In right triangle with legs a and b and hypotenuse c, the altitude divides the hypotenuse into segments with lengths of $\frac{a^2}{c}$ and $\frac{b^2}{c}$. The median divides the hypotenuse in half.

$\left|\frac{9}{5}-\frac{5}{2}\right| = \frac{7}{10}$, so 0.7. Pass: $\frac{X}{C} = \frac{7}{.7} = 10$. **Ans. 0.7, Pass 10**

Pink Relay – Seat D

If there is only one solution, the discriminate equals 0. $b^2 - 4(45)20 = b^2 - 3600 = 0$ and B = $\pm$ 60. The single solution is $\frac{-b}{90}$, b = -60, and the solution is $\frac{2}{3}$. Pass: $D^{-1}X = \frac{3}{2}(10) = 15$. **Ans. 2/3, Pass 15**

Pink Relay – Seat E

The altitude measures $\frac{ab}{c}$ and divides the hypotenuse into segments with lengths of $\frac{a^2}{c}$ and $\frac{b^2}{c}$. The median has length $\frac{c}{2}$. Plugging in the numbers, BD = $\frac{120}{17}$, BE = $\frac{17}{2}$, and DE = $\frac{17}{2}-\frac{64}{17}=\frac{289-128}{34}=\frac{161}{34}$. Perimeter = $\frac{161}{34}+\frac{120}{17}+\frac{17}{2} = \frac{161+240+289}{34} = \frac{690}{34} = 20\frac{5}{17}$. Pass: EX + 2E = $20\frac{5}{17}(15) + 2(20\frac{5}{17}) = 17(20\frac{5}{17}) = 345$. **Ans. $20\frac{5}{17}$, Pass 345**

Yellow Relay – Seat A

There are 7 choices for the first post and 6 remaining for the second. 7(6) = 42.
Pass: A/2 = 42/2 = 21. **Ans. 42, Pass 21**

Yellow Relay – Seat B

35(15) = 525 square feet. $\frac{525}{1\frac{5}{16}} = \frac{21\cdot 25\cdot 16}{21}$ = 400. The square has side length of 20.

Pass: BX = 20(21) = 420. **Ans. 20, Pass 420**

Yellow Relay – Seat C

As in Pink Relay C, $\left|\frac{36}{10}-\frac{10}{2}\right| = \frac{14}{10}$ = 1.4. Pass: $\frac{X}{C}=\frac{420}{1.4}$= 300. **Ans. 1.4, Pass 300**

Yellow Relay – Seat D

As in Pink Relay D, b^2 - 4(25)16 = b^2 - 1600 = 0. So b = ± 40. The single solution is $\frac{-b}{50}$, b = -40, and the solution is 4/5. Pass: $D^{-1}X = \frac{5}{4}(300)$ = 375. **Ans. 4/5, Pass 375**

Yellow Relay – Seat E

As in Pink Relay E, BD = $\frac{108}{15}$, BE = $\frac{15}{2}$, and DE = $\frac{15}{2}-\frac{81}{15} = \frac{225-162}{30}=\frac{63}{30}$. Perimeter = $\frac{63}{30}+\frac{216}{30}+\frac{225}{30} = \frac{504}{30} = 16\frac{4}{5}$. Pass: EX = $\frac{504}{30}\cdot 375 = \frac{84}{5}\cdot 375$ = 84(75) = 6300.

Ans. $16\frac{4}{5}$, Pass 6300

Answer Sheet – States 2014

Individuals 1
1. **36** or **36%**
2. **89**
3. **56/65**

Individuals 2
1. **84**
2. **5**
5. **2080**π

Individuals 3
1. **350** or **350 ft**
2. **x** $\geq$ **4**
3. **5005**

Individuals 4
1. **26**
2. **7**
3. **81** or **81**st

Individuals 5
1. **800**
2. **36**
3. **17**

Individuals 6
1. **969**
2. **30**
3. **264**$\sqrt{10\pi}$

Team 1
1. **34.6 or** $34\frac{3}{5}$
2. **9**
3. **Rhombus**
4. **560**
5. **-5**
6. **8**
7. **1879**
8. **5/13**

Team 2
1. **6**
2. **127**
3. **53**
4. **48**
5. **100**
6. **7**
7. **(0, -1)**
8. $\sqrt{3}/\pi$

	Blue Relay		Green Relay		Pink Relay		Yellow Relay	
	Answer	**Pass**	**Answer**	**Pass**	**Answer**	**Pass**	**Answer**	**Pass**
A	9	81	10	100	120	70	42	21
B	33	48	30	70	10	7	20	420
C	17	8	25	10	.7	10	1.4	300
D	2	17	-66	-7	2/3	15	4/5	375
E	1/34	578	1/442	-63	$20\frac{5}{17}$	345	$16\frac{4}{5}$	6300