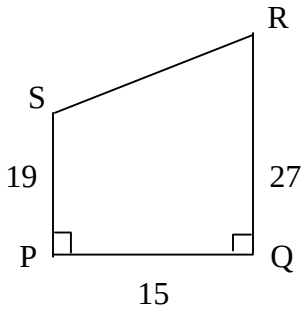
Round 1 Individuals States 2011

3 pts 1. Find the area of quadrilateral PQRS.



Ans. _____

4 pts 2. A store prices an item in dollars and cents so that when a 5% sales tax is added to the price, no rounding is necessary on the total cost, because the price of the item plus the tax results is exactly *n* dollars. Find the smallest value for *n*, where *n* is an integer.

Ans. _____

5 pts 3. The 7^{th} term of an arithmetic series is 59. The 19^{th} term is 131. If there are 61 terms in the series, find the sum of all the terms.

Ans. _____

Round 2 Individuals States 2011

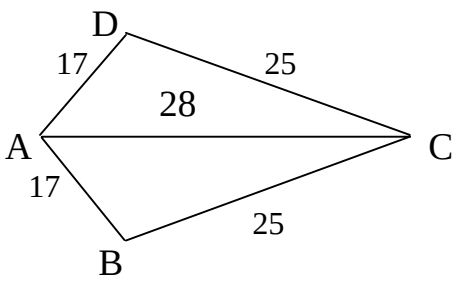
3 pts 1. On the number line, what number is halfway between $\frac{1}{7}$ and $\frac{1}{9}$?

Ans. _____

4 pts 2. If $f(x) = \frac{2}{3}x(x + 1)(x + 2)$, then find in simplest form $f(r) - f(r - 1)$.

Ans. _____

5 pts 3. Find the area of the kite ABCD.



Ans. _____

Round 3 Individuals States 2011

3 pts 1. Simplify: $16 \times 4 \div 4 \times 16$ **Ans. _____**

4 pts 2. Find the sum of the positive factors of 480.

Ans. _____

5 pts 3. A basketball is dropped from a height of 30 ft. and rebounds 2/3 of the height from which it was dropped. If it continues in this fashion, how many feet will the ball travel until it comes to rest?

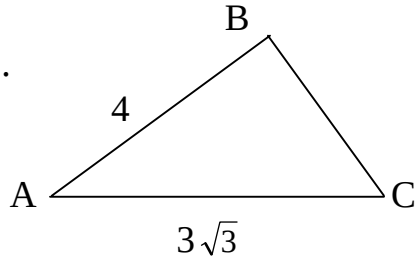
Ans. _____

Round 4 Individuals States 2011

3 pts 1. Before Andy started a 3-hour drive his odometer read 29792. When he reached his destination his odometer read another palindrome. If he never exceeded the speed limit of 75 mph, what was his greatest average speed in mph for the drive?

Ans. _____

4 pts 2. Find the length of side BC in the following triangle.
The measure of angle BAC is 30°, AB = 4 and AC = $3\sqrt{3}$.



Ans. _____

5 pts 3. Solve: $\frac{x-4}{x-3} - \frac{x-3}{x-1} = \frac{2x^2-20}{x^2-4x+3}$.

Ans. _____

Round 5 Individuals States 2011

3 pts 1. If $\frac{x^x y^y}{x^y y^x} = \left(\frac{y}{x}\right)^m$, find *m*.

Ans. _____

4 pts 2. At a soda machine, on the average, 2 out of every 3 persons choose a pepsi low-calorie soda. If 6 people come to get a drink from the machine, what is the probability that exactly 4 of them choose a pepsi low-calorie soda?

Ans. _____

5 pts 3. Find all values of x, such that

$$\log_3(x + 3) + \log_3(3x + 1) = \log_3(6x^2 - 3x - 7).$$

Ans. _____

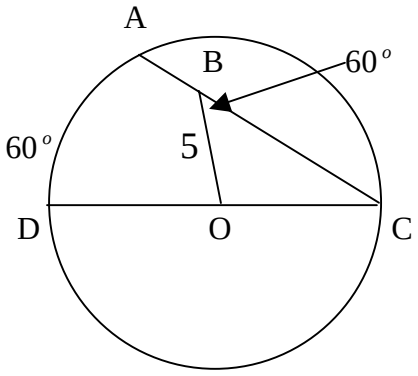
Round 6 Individuals States 2011

3 pts 1. Simpify: $\frac{2000^2}{1254^2 - 1246^2}$ **Ans.** _____

4 pts 2. Find the value of $\sin 105^o \cos 345^o - \cos 105^o \sin 345^o$

Ans. _____

5 pts 3. $\overline{DC}$ is the diameter through center O.
OB = 5, m$\angle$CBO = 60^o and measure arc AD = 60^o.
Find the length of segment AB.



Ans. _____

Round 1 Team States 2011

4 pts 1. Evaluate: $\dfrac{\log_3 \sqrt[5]{9} - \log_2 \sqrt[4]{8}}{\log_6 \sqrt[3]{36} + \log_5 \sqrt[6]{125}}$ **Ans.** _____

4 pts 2. In how many distinguishable ways can 2 apples from 5 apples and 2 oranges from 3 oranges be made? **Ans.** _____

6 pts 3. If $x - 2$ is a factor of $x^3 + kx^2 + 12x - 8$, find k. **Ans.** _____

6 pts 4. Find the vertices of the ellipse whose equation is

$16x^2 + 25y^2 - 96x + 200y + 144 = 0$. **Ans.** _____

6 pts 5. How many quarts of a solution which contains 50% salt must be mixed with 15 quarts a solution which is 90% salt, so as to make a solution which is 75% salt?

Ans. _____

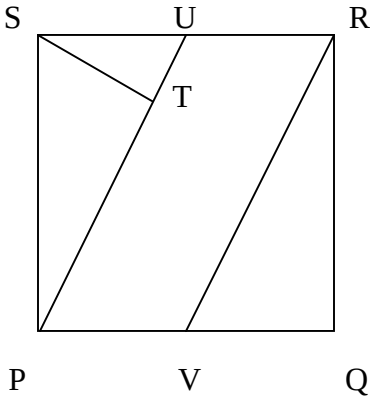
8 pts 6. If $-2^{-(2k+1)} + 2^{-(2k-1)} - 2^{-2k} = 2^P$, find P in simplest form.

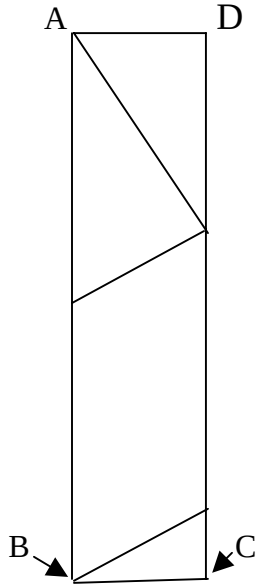
Ans. _____

8 pts 7. Let n be the number of ways that 10 dollars can be changed into dimes and quarters, with at least one of each coin being used. Find n.

Ans. _____

8 pts 8. Square PQRS is cut into 4 pieces, where U and V are midpoints and $\overline{ST} \perp \overline{UP}$. These four pieces are then put together to form rectangle ABCD. Find the ratio of the base BC to the height AB of the rectangle.





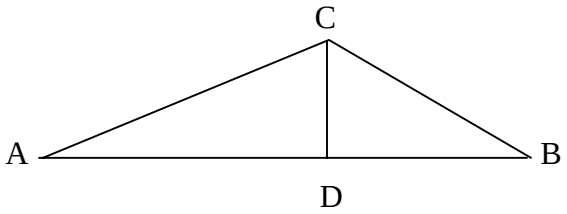
Ans. _____

Round 2 Team States 2011

4 pts 1. Simplify: $\dfrac{a-\dfrac{1}{b}}{b-\dfrac{1}{a}}$ **Ans.** _____

4 pts 2. In the fgure, $\overline{CD} \perp \overline{AB}$, AC = 863, $m\angle A = 29^o\,34'$, and $m\angle B = 42^o\,16'$. Find BC to the nearest 100^{th}.

Ans. _____



6 pts 3. Find the sum of the x and y-intercepts of the perpendicular bisector of $\overline{AB}$, where A = (5, 4) and B = (9, 2).

Ans. _____

6 pts 4. If it takes 60 men 200 hours to pick 5000 bushels of grapes, how many hours will it take 20 men to pick 1000 bushels?

Ans. _____

6 pts 5. Find all values of x such that: $2x^4 + 7x^3 + 14x^2 + 11x - 10 = 0$.

Ans. _____

8 pts 6. In $\triangle ABC$, $m\angle ABC = 120^o$, AB = 3 and BC = 4. If perpendiculars are constructed to $\overline{AB}$ at A and to $\overline{BC}$ at C meet at point D, find the length of $\overline{CD}$.

Ans. _____

8 pts 7. The first three terms of a geometric sequence of real numbers are x – 2, 2x – 2, and 8x – 2, where each succeeding term is greater than the previous term. What is the numerical sum of the first 5 terms of the sequence?

Ans. _____

8 pts 8. Watson's winning a game is in jeopardy. His chances of winning are enhanced greatly, if he can guess correctly some of the next 6 questions. If his probability of answering a question correctly is .8, what is the probability that he answers at least three of the 6 questions correctly? Give exact answer.

Ans. _____

Seat A Blue Relay States 2011

n nickels and *d* dimes make 65 cents. If there are 8 coins, how many nickels are there?

Pass back: 5A + 5 A = Your answer.

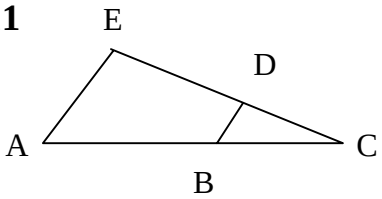
Seat B Blue Relay States 2011

Dan drove 7 hours at an average speed. Ed drove 6 hours at a speed which was 5 mph faster than Dan's, but covered 20 fewer miles. What was Dan's average speed?

Pass back: $\frac{2X+B}{2}$. B = Your answer. X = The number you will receive.

Seat C Blue Relay States 2011

$\overline{BD} \parallel \overline{AE}$, BD = 6, CD = 8, and DE = 12. Find AE.



Pass back: $\frac{X+3C}{3}$. C = Your answer. X = The number you will receive.

Seat D Blue Relay States 2011

Find the product of the roots of $3x^2 - x - 14 = 0$.

Pass back: $\frac{X-3D}{4}$. D = Your answer. X = The number you will receive.

Seat E Blue Relay States 2011

Evaluate: $2 \sin 75^o \cos 75^o$.

Pass in: $\frac{\frac{X}{2}+4E}{5}$. E = Your answer. X = The number you will receive.

Seat A Green Relay States 2011

Sam has 75 cents in nickels and dimes. If he has 11 coins in all, how many dimes does he have?

Pass back: 5A + 5. A = Your answer.

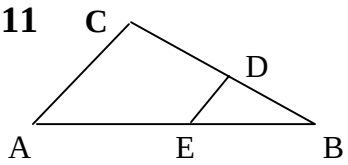
Seat B Green Relay States 2011

Pete drove 8 hours at an average speed. Rick drove 7 hours at an average speed 5 mph faster than Pete's speed and covered 5 fewer miles than Pete. How fast did Pete drive?

Pass back: $\frac{2X + B}{3}$. B = Your answer. X = The number you will receive.

Seat C Green Relay States 2011

$\overline{DE} \parallel \overline{AC}$, DE = 6, BD = 10, and CD = 15. Find AC.



Pass back: $\frac{3C + X}{5}$. C = your answer. X = The number you will receive.

Seat D Green Relay States 2011

Find the product of the roots of $4x^2 + 21x - 18 = 0$.

Pass back: $\frac{\frac{X}{2} - 3D}{7}$. D = Your answer. X = The number you will receive.

Seat E Green Relay States 2011

Evaluate: $\cos 75^o \cos 15^o + \sin 75^o \sin 15^o$.

Pass in: $\frac{2X + 6E}{9}$. E = Your answer. X = The number you will receive.

Seat A Pink Relay States 2011

If x < y, find the largest integral value for x that satisfies 5x + 2y < 47.

Pass back: 100A. A = Your answer.

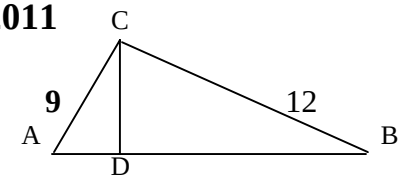
Seat B Pink Relay States 2011

Mr. Jones invested $20,000, part at 6% and the rest at 8%. If the 6% investment yielded $80 more than the 8% investment, how much was invested at 6%?

Pass back: $\frac{10B}{X}$ B = Your answer. X = The number you will receive.

Seat C Pink Relay States 2011

$m\angle ACB = 90^{o}$, AC = 9, BC = 12 and $\overline{CD} \perp \overline{AB}$, find CD.



Pass back: $\frac{CX}{20}$ C = Your answer. X = The number you will receive.

Seat D Pink Relay States 2011

Find the smallest integer x such that $|4x-8| \le 7x+3$.

Pass back: $\frac{X}{D+8}$ D = Your answer. X = The number you will receive.

Seat E Pink Relay States 2011

In how many distinguishable ways can the letters of "Statemeet" be rearranged?

Pass in: $\frac{E}{9X}$ E = Your answer. X = The number you will receive.

Seat A Yellow Relay States 2011

If $x < y$ find the largest integral value of x such that $8x + 5y < 81$.

Pass back: A + 14. A = Your answer.

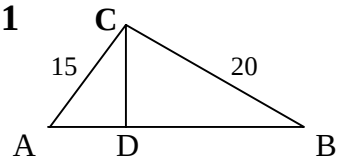
Seat B Yellow Relay States 2011

Mr. Jones invested $20,000, part at 6% and the rest at 8%. If the 6% investment yielded $80 more than the 8% investment, how much was invested at 8%?

Pass back: $\frac{B}{2X}$ B = Your answer. X = The number you will receive.

Seat C Yellow Relay States 2011

$\overline{AC} \perp \overline{BC}$, $\overline{CD} \perp \overline{AB}$, AC = 15 and BC = 20. Find CD.



Pass back: $\frac{3X}{10C}$ C = Your answer. X = The number you will receive.

Seat D Yellow Relay States 2011

Find the least integer x, such that $|5x-12| \le 3x+2$.

Pass back: 10X + 9D D = Your answer. X = The number you will receive.

Seat E Yellow Relay States 2011

In how distinguishable many ways can the letters of "statesday" be rearranged?

Pass in: $\frac{E}{X+2}$ E = Your answer. X = The number you will receive.

Solutions – Individuals Round 1

1. Trapezoid area: ½ (15)(19 + 27) = 15(23) = 345. **Ans. 345**

2. Let n = price. Then 05n = 1, n = 20. Selling $21. **Ans. 21**

3. 59 = a + 6d and 131 = a + 18d. Subtracting: 72 = 12d ➔ d = 6. Subbing back in: 59 = a + 6(6) ➔ 23 = a. 61st term = 23 + 60(6) = 383. Sum = $\frac{61}{2}(23 + 383)$ ➔ 61(406)/2 = 61(203) = 12,383. **Ans. 12,383**

Individuals Round 2

1. $\frac{1}{2}\left(\frac{1}{7}+\frac{1}{9}\right) = \frac{1}{2}\left(\frac{16}{63}\right) = \frac{8}{63}$. **Ans. 8/63**

2. f(r) = $\frac{2}{3}$r(r + 1)(r + 2), f(r – 1) = $\frac{2}{3}$(r – 1)(r)(r + 1). f(r) - f(r – 1) = $\frac{2}{3}$r(r + 1)[r + 2 – (r – 1)] = $\frac{2}{3}$r(r + 1)(3) = 2r(r + 1). **Ans. 2r(r + 1)**

3. Students familiar with the Pythagorean triplets may consider that the diagonals are both whole numbers. If so the other diagonal of the kite would have to be in an 8-15-17 right triangle. That would mean that the other triangle of the kite would have to be a 15-20-25 right triangle, which it is. So the area is ½ the product of the diagonals ➔ ½ (28)(30) = 420. Otherwise they could use Heron's Formula for each triangle and double it: $\sqrt{35(7)(10)(18)} = \sqrt{5\cdot7\cdot7\cdot5\cdot2\cdot2\cdot9} = 5\cdot7\cdot2\cdot3 = 210$. Doubled = 420. **Ans. 420**

Individuals Round 3

1. Order of operations is to multiply and divide left to right: $(16\times4)\div4\times16$ ➔ $(64\div4)\times16$ ➔ $16\times16 = 256$. **Ans. 256**

2. As you express 480 as a product of two numbers, using the left number as the smaller and the right as the larger of the two factors. The numbers on the left get larger as the numbers on the right get smaller, until they are both equal or almost so. Thus: 1(480), 2(240), 3(160), 4(120), 5(96), 6(80), 8(60), 10(48), 12(40), 15(32), 16(30), 20(24). Adding the left numbers is 102, adding the right numbers is 1410. Sum = 1512. **Ans. 1512**

3. The down sequence is 30 + 20 + 20/3 … . The up sequence is 20 + 20/3 + 40/9 … . Sum = $30 + 2\left(\frac{20}{1-\frac{2}{3}}\right) = 30 + 2(60) = 150$. **Ans. 150**

Individuals Round 4

1. The next three palindromes are 29892, 29992, 30003. 30003 ➔ $70\frac{1}{3}$ mph. **Ans. $70\frac{1}{3}$**

2. Dropping a perpendicular from B to meet $\overline{AC}$ at D makes a 30-60-90 Δ where BD = 2 and AD = $2\sqrt{3}$. Thus DC = $\sqrt{3}$. In triangle BDC, which is a right triangle, using the Pythagorean Theorem, BC = $\sqrt{7}$. **Ans.** $\sqrt{7}$

3. $\frac{x-4}{x-3}-\frac{x-3}{x-1}=\frac{2x^2-20}{x^2-4x+3}$ ➔ (x – 4)(x – 1) – (x - 3)(x – 3) = $2x^2$ - 20 ➔
x^2 – 5x + 4 – (x^2 – 6x + 9) = $2x^2$ - 20 ➔ x – 5 = $2x^2$ - 20. Thus 0 = $2x^2$ - x – 15 or 0 = (2x + 5)(x – 3). Thus x = 3 or -2½. But x cannot be 3. **Ans. -2½**

Individuals Round 5

1. $\frac{x^x y^y}{x^y y^x}=\left(\frac{y}{x}\right)^m$ ➔ $\frac{y^y y^{-x}}{x^y x^{-x}}=\frac{y^{y-x}}{x^{y-x}}=\left(\frac{y}{x}\right)^{y-x}=\left(\frac{y}{x}\right)^m$. So *m* = y – x. **Ans. y – x**

2. $pf \cdot pf \cdot pf \cdot pf \cdot \cancel{pf} \cdot \cancel{pf} \left(\frac{6!}{4!2!}\right) = \left(\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\right)(15) = \frac{80}{243}$. **Ans. 80/243**

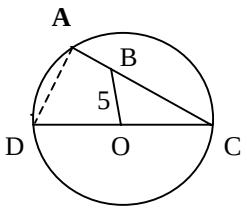
3. $\log_3(x+3)+\log_3(3x+1)=\log_3(6x^2-3x-7)$ ➔ (x + 3)(3x + 1) = $6x^2$ - 3x – 7 ➔
$3x^2$ + 10x + 3 = $6x^2$ - 3x – 7 ➔ 0 = $3x^2$ - 13x – 10 ➔ 0 = (3x + 2)(x – 5).
Thus x = 5 or -2/3. But -2/3 cannot be used. **Ans. 5**

Individuals Round 6

1. $\frac{2000^2}{1254^2-1246^2} = \frac{2000\cdot 2000}{(1254+1246)(1254-1246)} = \frac{2000\cdot 2000}{2500\cdot 8} = \frac{4\cdot 2000}{5\cdot 8} = 4\cdot 50 = 200$. **Ans. 200**

2. $\sin 105^o \cos 345^o - \cos 105^o \sin 345^o = \sin(-240) = \sqrt{3}/2$. **Ans.** $\sqrt{3}$**/2**

3. Since $\angle C = 30^o$, then ΔBOC is a 30-60-90 Δ, thus BC = 10.
Connecting A to D, makes m$\angle A = 90^o$, so ΔADC is also a 30-60-90 Δ. Since OC = $5\sqrt{3}$, then DC = $10\sqrt{3}$, and AD = $5\sqrt{3}$.
Thus AC = 15, and since BC = 10, AB = 5. **Ans. 5**



Round 1 Team

1. $\frac{\log_3\sqrt[5]{9}-\log_2\sqrt[4]{8}}{\log_6\sqrt[3]{36}+\log_5\sqrt[6]{125}} = \frac{\frac{2}{5}-\frac{3}{4}}{\frac{2}{3}+\frac{1}{2}} = \frac{\frac{8}{20}-\frac{15}{20}}{\frac{4}{6}+\frac{3}{6}} = \frac{-\frac{7}{20}}{\frac{7}{6}} = -\frac{7}{20}\cdot\frac{6}{7} = -\frac{3}{10}$ **Ans. – 3/10**

2. ${}_5C_2 \cdot {}_3C_2 = 10(3) = 30$. **Ans. 30**

3. $x - 2 \,)\, x^3 + kx^2 + 12x - 8$

$x^2 + (k+2)x + (2k+16)$

$-x^3 + 2x^2$

$(k+2)x^2 + 12x$

$-(k+2)x^2 + 2(k+2)x$

$(2k+16)x - 8$

$-(2k+16)x + 2(2k+16)$

$4k + 32 - 8 = 0 \rightarrow 4k + 24 = 0.\ k = -6.$ **Ans. -6**

4. $16x^2 + 25y^2 - 96x + 200y + 144 = 0 \rightarrow 16(x^2 - 6x + 9) + 25(y^2 + 8y + 16) = 400$

$\frac{(x-3)^2}{25} + \frac{(y+4)^2}{16} = 1$. Center is (3, -4), vertices are at $(3 \pm 5, -4)$ **Ans. (8, -4), (-2, -4)**

5. $.50x + .90(15) = .75(x + 15) \rightarrow 50x + 90(15) = 75x + 75(15) \rightarrow 15(15) = 25x.$
Thus x = 9. **Ans. 9**

6. $-2^{-(2k+1)} + 2^{-(2k-1)} - 2^{-2k} = 2^P \rightarrow -2^{-2k-1} + 2^{-2k+1} - 2^{-2k} = 2^p \rightarrow -\frac{1}{2}\left(2^{-2k}\right) + 2\left(2^{-2k}\right) - 2^{-2k}$

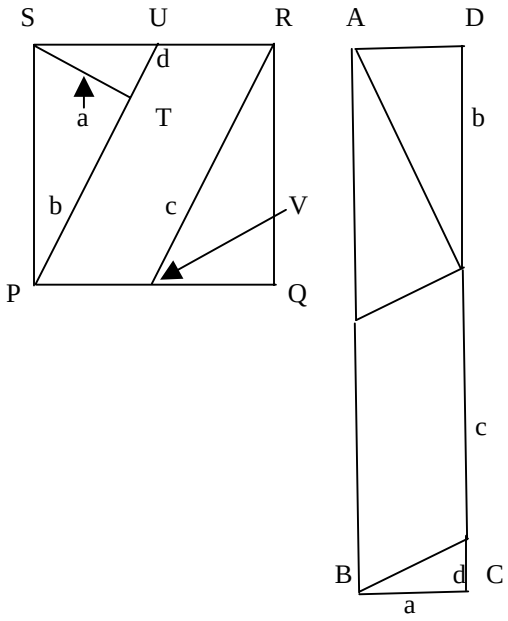
$(2^{-2k})(-\frac{1}{2} + 2 - 1) = (2^{-2k})(1/2) = 2^{-2k-1} = 2^p$. So p = -2k – 1. **Ans. -2k – 1**

7. There must be at least 5 dimes, and multiples of 5 dimes. Thus .50, 1.00, 1.50, …, but can be no more than 9.50 or else it would be all dimes. So $950 = 50 + (n - 1)\ 50 \rightarrow$ $900 = (n - 1)\ 50 \rightarrow 18 = n - 1$. So n = 19. **Ans. 19**

8. Let SP = 2. The three right triangles are all similar, with side ratio's of 1, 2, $\sqrt{5}$. So a = $\frac{2}{\sqrt{5}}$, b = $\frac{4}{\sqrt{5}}$, d = $\frac{1}{\sqrt{5}}$ and c = $\sqrt{5}$. In rectangle ABCD, CB = $\frac{2}{\sqrt{5}}$, and AB = DC = b + c + d = $\frac{4}{\sqrt{5}} + \sqrt{5} + \frac{1}{\sqrt{5}} =$

$\frac{4}{\sqrt{5}} + \frac{5}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \frac{10}{\sqrt{5}}$. The ratio of $\frac{BC}{AB} = \frac{\frac{2}{\sqrt{5}}}{\frac{10}{\sqrt{5}}} = \frac{1}{5}$.

Ans. 1/5



Round 2 Team

1. $\dfrac{a-\frac{1}{b}}{b-\frac{1}{a}} = \dfrac{\frac{ab-1}{b}}{\frac{ab-1}{a}} = \dfrac{ab-1}{b}\cdot\dfrac{a}{ab-1} = \dfrac{a}{b}$. **Ans. a/b**

2. $\sin 29^o 34' = \dfrac{CD}{863}$, so CD = 863 $\sin 29^o 34'$. $\sin 42^o 16' = \dfrac{CD}{BC}$, so BC = $\dfrac{CD}{\sin 42^o 16'}$.

Thus BC = $\dfrac{863\sin 29^o 34'}{\sin 42^o 16'}$ = 633.1349. **Ans. 633.13**

3. The midpoint of (5, 4) and (9, 2) is (7, 3). Slope through the two points: $\dfrac{4-2}{5-9} = -\dfrac{1}{2}$. Slope of perpendicular bisector = 2. Equation form y = 2x + b, or 2x – y = c. Plugging in (7, 3): 2(7) – 3 = c. Thus the equation is 2x – y = 11. x-i = 5½, y-i = -11. **Ans. – 5 1/2**

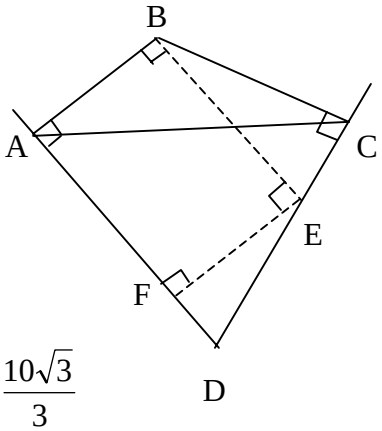
4. $k = \dfrac{men \cdot hours}{bushels}$ since if the men are increased while the number of bushels stays the same, then the number of hours will decrease. Therefore men and hours are inversely proportional. Keeping the hours the same, if the men are increased the number of bushels also increases, a direct proportion. Thus $\dfrac{60\cdot 200}{5000} = \dfrac{20h}{1000}$ ➔ $60\cdot 200 = 100h$. **Ans. 120**

5. -2 | 2 7 14 11 -10 Taking ½ of 2 4 10: 1 2 5 produces $x^2 + 2x + 5 = 0$.
-4 -6 -16 10

1/2 | 2 3 8 -5 0 By completing the square: $x^2 + 2x + 1 = -5 + 1$.
1 2 5

2 4 10 Thus $(x + 1)^2 = -4$ ➔ $x + 1 = \pm 2i$, $x = -1 \pm 2i$. **Ans. ½ , -2, -1 ± 2i**

6. In the figure, since m∠ABC = 120^o, then m∠D = 60^o because the sum of the angles of a quadrilateral is 360^o. Dropping a perpendicular from B to $\overline{DC}$ at E and then dropping a perpendicular from E to $\overline{AD}$ at F, forms two 30-60-90∆'s, ∆BDC and ∆EFD. Since BC =4, then CE = $\dfrac{4}{\sqrt{3}}$. EF = AB because they are opposite sides of a rectangle, then DE = $\dfrac{6}{\sqrt{3}}$. Thus CD = $\dfrac{10}{\sqrt{3}}$ or $\dfrac{10\sqrt{3}}{3}$. **Ans.** $\dfrac{10\sqrt{3}}{3}$



7. The 2nd term divided by the first will give common ratio, so: $\dfrac{2x-2}{x-2} = \dfrac{8x-2}{2x-2}$, thus $(2x – 2)(2x – 2) = (x – 2)(8x – 2)$ ➔ $4x^2 - 8x + 4 = 8x^2 - 18x + 4$ ➔ $0 = 4x^2 - 10x$. Factoring: 2x(2x – 5) = 0. So x = 0 or 2½. Since each succeeding term is greater the previous, x = 2½. Subbing in, terms are ½ , 3, 18, 108, 648. Sum = 777½. **Ans. 777½**

8. $C \cdot C \cdot C \cdot \not{C} \cdot \not{C} \cdot \not{C} \left(\frac{6!}{3!3!}\right) + C \cdot C \cdot C \cdot C \cdot \not{C} \cdot \not{C} \left(\frac{6!}{4!2!}\right) + C \cdot C \cdot C \cdot C \cdot C \cdot \not{C} \left(\frac{6!}{5!1!}\right) + C \cdot C \cdot C \cdot C \cdot C \cdot C =$

$(.8)^3(.2)^3(20) + (.8)^4(.2)^2(15) + (.8)^5(.2)(6) + (.8)^6 =$
$.081920 + .245760 + .393216 + .262144 = .98304.$ **Ans. .98304**

Seat A Blue Relay

Let n nickels and (8 – n) dimes. Then 5n + 10(8 – n) = 65. 5n + 80 – 10n = 65.
-5n = -15. n = 3. Pass: 5A + 5 = 5(3) + 5 = 20. **Ans. A = 3, Pass: 20**

Seat B Blue Relay

Dan R(7) = D
Ed (R + 5)(6) = D – 20 ➔ 6R + 30 = 7R – 20 ➔ 50 = R, X = 20.
Pass back: $\frac{2X+B}{2} = \frac{2(20)+50}{2} = \frac{90}{2}$ = 45. **Ans. B = 50, Pass: 45**

Seat C Blue Relay

$\frac{8}{20} = \frac{6}{x}$ ➔ 8x = 120, x = 15. Pass: $\frac{X+3C}{3} = \frac{45+3(15)}{3} = \frac{90}{3}$ = 30. **Ans. C = 15, Pass: 30**

Seat D Blue Relay

$3x^2$ - x – 14 = 0. Product = -14/3. Pass: $\frac{X-3D}{4} = \frac{30-3(-14/3)}{4} = \frac{30+14}{4} = 11$

Ans. D = -14/3, Pass: 11

Seat E Blue Relay

$2 \sin 75^o \cos 75^o = \sin 150^o = ½$. Pass: $\frac{\frac{X}{2}+4E}{5} = \frac{\frac{11}{2}+4\left(\frac{1}{2}\right)}{5} = \frac{\frac{15}{2}}{5} = \frac{3}{2}$. **Ans. E = $\frac{1}{2}$, Pass: $\frac{3}{2}$**

Seat A Green Relay

Let d dimes and (11 – d) nickels. 5(11 – d) + 10d = 75 ➔ 55 – 5d + 10d = 75 ➔ 5d = 20, so d = 4. Pass: 5A + 5 = 5(4) + 5 = 25. **Ans. A = 4, Pass: 25**

Seat B Green Relay

Pete R(8) = D
Rick (R + 5)7 = D – 5 ➔ 7R + 35 = 8R – 5 ➔ 40 = R.
Pass back: $\frac{2X+B}{3} = \frac{2(25)+40}{3} = \frac{90}{3}$ = 30. **Ans. B = 40, Pass: 30**

Seat C Green Relay

$\frac{10}{25} = \frac{6}{x}$ ➔ 10x = 150 , x = 15. Pass: $\frac{3C+X}{5} = \frac{3(15)+30}{5} = \frac{75}{5}$ = 15 **Ans. C = 15, Pass: 15**

Seat D Green Relay

$4x^2 + 21x - 18 = 0$, Product = - 18/4 = - 4½. Pass: $\frac{\frac{X}{2} - 3D}{7} = \frac{\frac{15}{2} - 3\left(\frac{-9}{2}\right)}{7} = \frac{\frac{42}{2}}{7} = 3.$

Ans. -4½, Pass: 3

Seat E Green Relay

$\cos 75^o \cos 15^o + \sin 75^o \sin 15^o = \cos(75^o - 15^o) = \cos 60^o = ½.$

Pass: $\frac{2X + 6E}{9} = \frac{2(3) + 6(1/2)}{9} = \frac{9}{9} = 1.$ **Ans. E = ½ , Pass: 1**

Seat A Pink Relay

(1) 5x + 2y < 47, (2) x < y ➔ 2x – 2y < 0. Adding (1) and (2): 7x < 47, so $x < 6\frac{5}{7}$, so largest integral value for x is 6. Pass: 100A = 100(6) = 600. **Ans. A = 6, Pass: 600**

Seat B Pink Relay

.06x = .08(20,000 – x) + 80 ➔ 6x = 160,000 – 8x + 8,000 ➔ 14x = 168,000, x = 12,000.

Pass: $\frac{10B}{X} = \frac{10(12{,}000)}{600} = 200.$ **Ans. B = 12,000, Pass: 200**

Seat C Pink Relay

AB = 15 and by area: 9(12) = 15h, so h = $\frac{108}{15} = 7\frac{1}{5}$ or 7.2. Pass: $\frac{CX}{20} = \frac{7.2(200)}{20} = 7.2(10)$

Ans. C = 7.2, Pass: 72

Seat D Pink Relay

$|4x - 8| \le 7x + 3$ ➔ Critical points are at (1) 4x – 8 = 7x + 3 and (2) 4x – 8 = -7x – 3

In (1): -11 = 3x, so x = -3⅔. In (2): 11x = 5, so x = 5/11. Only numbers in the interval x ≥ 5/11 work. So smallest integer is 1. Pass: $\frac{X}{D+8} = \frac{72}{1+8} = 8.$ **Ans. D = 1, Pass: 8**

Seat E Pink Relay

$\frac{9!}{3!3!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3 \cdot 2 \cdot 3!} = 9 \cdot 8 \cdot 7 \cdot 5 \cdot 4 = 72 \cdot 140 = 10,080.$ Pass: $\frac{E}{9X} = \frac{10{,}080}{9(8)} = \frac{72(140)}{9(8)} = 140.$

Ans. E = 10,080, Pass: 140

Seat A Yellow Relay

(1) 8x+ 5y < 81 and (2) x < y. In (2) 5x – 5y < 0. Adding this to (1): 13x < 81, so $x < 6\frac{3}{13}$. Thus the largest integer for x is 6. Pass: A + 14 = 20. **Ans. A = 6, Pass: 20**

Seat B Yellow Relay

Answer from Pink B - \$8,000. Pass: $\frac{B}{2X} = \frac{8000}{2(20)} = 200.$ **Ans. B = 8,000, Pass: 200**

Seat C Yellow Relay

AB = 25. So through area: 15(20) = 25h ➔ h = 12. Pass: $\frac{3X}{10C} = \frac{3(200)}{10(12)} = 5$

Ans. C = 12, Pass: 5

Seat D Yellow Relay

$|5x-12| \le 3x+2$ ➔ Critical points are at (1) 5x – 12 = 3x + 2 and at (2) 5x – 12 = - 3x – 2. In (1): 2x = 14, x = 7. In (2): 8x = 10, x = 1¼ . Only the interval 1 ¼ ≤ x ≤ 7 works. The least integer is 2. Pass: 10X + 9D = 10(5) + 9(2) = 68. **Ans. D = 2, Pass: 68**

Seat E Yellow Relay

$\frac{9!}{2!2!2!} = 9 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 63(720) = 45{,}360.$ Pass: $\frac{E}{X+2} = \frac{63(720)}{68+2} = \frac{63(720)}{70} = 9(72) = 648.$

Ans. 45,360, Pass: 648

Answer Sheet States 2011

Round 1 Individuals

1. **345**
2. **21**
3. **12,383**

Round 2 Individuals

1. **8/63**
2. **2r(r + 1)** or $2r^2 + 2r$
3. **420**

Round 3 Individuals

1. **256**
2. **1512**
3. **150** or **150 ft**

Round 4 Individuals

1. $70\frac{1}{3}$ **m/h** or $70\frac{1}{3}$ **(211/3** or $70.\overline{3}$**)**
2. $\sqrt{7}$
3. **– 5/2** or **– 2½** or **– 2.5**

Round 5 Individuals

1. **y – x**
2. **80/243**
3. **5**

Round 6 Individuals

1. **200**
2. $\sqrt{3}$**/2**
3. **5**

Round 1 Team

1. **– 3/10** or **– .3**
2. **30**
3. **– 6**
4. **(8, – 4), (– 2, – 4)**
5. **9** or **9 quarts**
6. **– 2k – 1**
7. **19**
8. **1/5**

Round 2 Team

1. **a/b**
2. **633.13**
3. **-5½** or **– 11/2** or **– 5.5**
4. **120** or **120 hr**
5. **1/2, – 2, –1** $\pm$ **2i**
6. $\frac{10\sqrt{3}}{3}$
7. **777½** or **777.5** or **1555/2**
8. **.98304**

Relays

Blue	Ans.	Pass	Green	Ans.	Pass	Pink	Ans.	Pass	Yellow	Ans.	Pass
A	3	20	A	4	25	A	6	600	A	6	20
B	50	45	B	40	30	B	12,000	200	B	8,000	200
C	15	30	C	15	15	C	7.2	72	C	12	5
D	–14/3	11	D	– 4½	3	D	1	8	D	2	68
E	1/2	3/2	E	1/2	1	E	10,080	140	E	45,360	648