

## 1 Individuals States 2026

**3 pts 1.** Triangle ABC is similar to triangle DEF.  $AB = 8$  cm,  $BC = 12$  cm,  $AC = 16$  cm and  $DE = 12$  cm. Find the perimeter of triangle DEF in cm.

Ans. \_\_\_\_\_

**4 pts 2.** What is the unit's digit of the sum  $1 + 2 + 3 + \dots + 2027$ ?

Ans. \_\_\_\_\_

**5 pts 3.** Compute the following:  $9778894 \cdot 9778896 + 9778896 \cdot 9778899 - 9778895 \cdot 9778898 - 9778898 \cdot 9778894$

Ans. \_\_\_\_\_

## 2 Individuals States 2026

**3 pts 1.** Suppose 15% of  $x$  equals 20% of  $y$ , and  $x$  and  $y$  are greater than 0.  $y$  is what percent of  $x$ ?

Ans. \_\_\_\_\_

**4 pts 2.** Solve for the matrix  $X$ , if  $X + 2 \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} = -\frac{1}{3} \begin{bmatrix} -6 & 0 \\ 9 & -3 \end{bmatrix}$ .

Ans. \_\_\_\_\_

**5 pts 3.** If two of the roots of the equation  $x^3 + bx^2 + cx + d = 0$  are  $-1$  and  $-3i$ , where  $b$ ,  $c$  and  $d$  are integers, find the value of  $b + c + d$ .

Ans. \_\_\_\_\_

### 3 Individuals States 2026

**3 pts 1.** Given that  $f(x) = 4.8 + 0.8x$  and  $g(x) = 0.56x - 2.4$ , what is the solution for  $f(x) = g(x)$ ?

**Ans.** \_\_\_\_\_

**4 pts 2.** Find the remainder when  $2x^5 - 36$  is divided by  $x - 2$ .

**Ans.** \_\_\_\_\_

**5 pts 3.** A rhombus has diagonals with lengths of  $2x$  and  $2x + 2$ . If its perimeter is  $4x + 36$ , find  $x$ .

**Ans.** \_\_\_\_\_

### 4 Individuals States 2026

**3 pts 1.** A game spinner is divided into 24 equal sectors. Some sectors are blue, some are purple, some are orange, some are red and some are green. Each of the sectors have equal areas, so when someone spins, the arrow has an equal likelihood of stopping on any sector. If the probability of landing on a blue sector, represented by  $P(\text{blue})$ , is  $1/8$ , and if  $P(\text{purple})$  is  $1/24$ ,  $P(\text{orange})$  is  $1/2$  and  $P(\text{red})$  is  $1/4$ , what is the probability that the arrow will land on green?

**Ans.** \_\_\_\_\_

**4 pts 2.** Find the value  $k$  so that the line  $(2 + k)x + (2 - k)y = 15$  has a slope of  $3/2$ .

**Ans.** \_\_\_\_\_

**5 pts 3.** Find the real solution(s) for the equation  $\sqrt{14 + \sqrt{27 - \sqrt{x - 1}}} = 4$ .

**Ans.** \_\_\_\_\_

## 5 Individuals States 2026

**3 pts 1.** Solve for  $x$ :  $\frac{1}{2}x + \frac{2}{3} = \frac{3}{4}x + \frac{4}{5}$

**Ans.** \_\_\_\_\_

**4 pts 2.** How many integral values of  $x$  are there such that  $|x^3 - 3| < 51$ .

**Ans.** \_\_\_\_\_

**5 pts 3.** Find all  $\theta$ , where  $0 < \theta < 2\pi$ , and  $\cos(-\theta) = \sin(-\theta)$ . State answer only in radians.

**Ans.** \_\_\_\_\_

## 6 Individuals States 2026

**3 pts 1.** The base of a right prism is a regular octagon. The area enclosed by the octagon is 26. The altitude of the prism is 15. Find the volume of the prism.

**Ans.** \_\_\_\_\_

**4 pts 2.** Find  $x + y + z$ , if  $2x - 6y + 7z = 16$

$$x - 2y + 4z = 10$$

$$3x - 5y + 8z = 23$$

**Ans.** \_\_\_\_\_

**5 pts 3.** An ellipse has a center at  $(7, 7)$ , a vertex at  $(17, 7)$  and a focus at  $(15, 7)$ . Find the equation of the ellipse in the form  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ .

**Ans.** \_\_\_\_\_

## 1 Team States 2026

**4 pts 1.** Solve the following for  $a$ :  $\log(a^2 - 15a) = 2$ .

(1) Ans. \_\_\_\_\_ 4 pts

**4 pts 2.** The median of a set of 15 consecutive integers is 42. What is the greatest of these 15 integers?

(2) Ans. \_\_\_\_\_ 4 pts

**6 pts 3.** Find the surface area of a cube in square inches if its volume is 1728 cubic inches.

(3) Ans. \_\_\_\_\_ 6 pts

**6 pts 4.** In triangle RST, with right angle at S,  $\sin T = \frac{x}{x+8}$  and  $\cos T = \frac{x+7}{x+8}$ . Find the value of  $\tan T$ .

(4) Ans. \_\_\_\_\_ 6 pts

**6 pts 5.** In the expansion of  $\left(4x^3 - \frac{2}{x^2}\right)^5$  one of the terms is a constant. Find the value of that constant term.

(5) Ans. \_\_\_\_\_ 6 pts

**8 pts 6.** A collection of recycled glass bottles consists of clear, brown and green bottles. The ratio of clear to brown to green is given by  $\frac{1}{2} : \frac{2}{5} : \frac{8}{9}$ . There are 483 bottles in total. What is the total number of the clear and green glass bottles combined?

(6) Ans. \_\_\_\_\_ 8 pts

**8 pts 7.** 524,288 is which term in the expansion of the following sequence?

2, 8, 32, 128, ...

(7) Ans. \_\_\_\_\_ 8 pts

**8 pts 8.** Each of the basic six trig functions ( $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $\cot x$ ,  $\sec x$  and  $\csc x$ ) is written on its own slip of paper. These slips of paper are placed in a hat, from which two slips are drawn at random. What is the probability that when the two slips are drawn, and the function on each slip is squared and added, the result will be equal to 1?

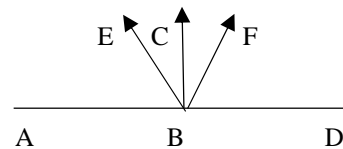
(8) Ans. \_\_\_\_\_ 8 pts

## 2 Team States 2026

**4 pts 1.** The volume of a cone with non-zero height is given by  $V = \frac{1}{3} \pi r^2 h$ . Find the radius ( $r$ ) in terms of  $V$  and  $h$ .

**(1) Ans.** \_\_\_\_\_ **4 pts**

**4 pts 2.**  $\overline{BC} \perp \overline{AD}$  and  $\overline{BC}$  bisects  $\angle EBF$ . If  $m\angle EBC = 5x - 36$ ,  $m\angle CBF = 4x - 7y$  and  $m\angle FBD = 3x + 2y$ , find the value of  $x + y$ .



**(2) Ans.** \_\_\_\_\_ **4 pts**

**6 pts 3.** Solve for  $x$ , if  $\begin{vmatrix} 1 & x & 3 \\ 4 & 5 & 6 \\ -1 & -x & 0 \end{vmatrix} = -9$ .

**(3) Ans.** \_\_\_\_\_ **6 pts**

**6 pts 4.** Solve for  $y$ :  $-\frac{20}{3} \leq -\frac{2}{3} - 2y \leq \frac{10}{3}$ . Give answer in the form  $a \leq y \leq b$ .

**(4) Ans.** \_\_\_\_\_ **6 pts**

**6 pts 5.** Find the value of  $\cos(x + y)$ , if  $\tan x = \frac{3}{4}$ , where  $0 \leq x < \frac{\pi}{2}$ , and  $\sin y = -\frac{5}{13}$ , where  $\pi \leq y < \frac{3\pi}{2}$ .

**(5) Ans.** \_\_\_\_\_ **6 pts**

**8 pts 6.** A circle is inscribed in a right triangle with legs of 7 and 24. Find the area of the circle.

**(6) Ans.** \_\_\_\_\_ **8 pts**

**8 pts 7.** Solve the following for  $x$ :  $0.05(x + 3) - 0.25(x + 4) = 0.125(x - 5) - 0.625(x - 6)$ .

**(7) Ans.** \_\_\_\_\_ **8 pts**

**8 pts 8.** If the sum of all the interior angles except one of a convex polygon is 2026 degrees, what must the number of sides of the polygon be?

**(8) Ans.** \_\_\_\_\_ **8 pts**

**Seat A Blue Relay States 2026**

$21x^2 + 22x - 24 = (ax + b)(cx + d)$ .  $a > c > 0$ ,  $a, b, c, d$  are integers. Find the value of  $a+b+c+d$ .

Pass Back:  $A - 6$

$A =$  Your Answer

**Seat B Blue Relay States 2026**

Harold has three children. The sum of the ages of the two youngest children is nine more than that of the oldest child. The sum of the two oldest children's ages is 13 more than the age of the youngest child. How old is the middle child?

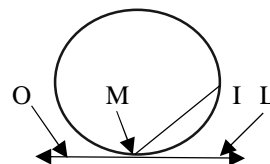
Pass Back:  $B^{X-B}$

$B =$  Your Answer

$X =$  The number you will receive

**Seat C Blue Relay States 2026**

$\overline{OL}$  is tangent to the circle at point M.  $\overline{MI}$  is a chord of the circle with  $m\widehat{MI} = 78^\circ$ . Find  $m\angle IMO$ .



Pass back:  $10C - X$

$C =$  Your Answer

$X =$  The number you will receive

**Seat D Blue Relay States 2026**

The perpendicular bisector of the segment with end points  $(3, 6)$  and  $(5, 8)$ , is written in the form  $Ax + By = C$ .  $A, B$ , and  $C$  are relatively prime and  $A > 0$ . Find the value of  $A + B + C$ .

Pass back:  $X + D + 3$

$D =$  Your Answer

$X =$  The number you will receive

**Seat E Blue Relay States 2026**

If  $a \# b = aba^b + bab^a$ . Find the value of  $(3 \# 2)$ .

Pass in:  $3X - 2E$

$E =$  Your answer

$X =$  The number you will receive

**Seat A Green Relay States 2026**

$32x^2 - 52x + 15 = (fx + g)(hx + j)$ , for  $f > h > 0, f, g, h, j$  are integers. Find the value of  $f + g + h + j$ .

Pass Back:  $A^A$

A = Your Answer

**Seat B Green Relay States 2026**

Harold has three children. The sum of the ages of the two youngest children is 10 more than that of the oldest child. The sum of the two oldest children's ages is 14 more than the age of the youngest child. How old is the middle child?

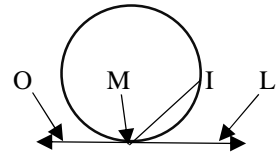
Pass back:  $\frac{3X}{B+4} + 5$

B = Your answer

X = The number you will receive

**Seat C Green Relay States 2026**

$\overline{OL}$  is tangent to the circle at point M.  $\overline{MI}$  is a chord of the circle with  $m\widehat{MI} = 64^\circ$ . Find  $m\angle IMO$ .



Pass back:  $3X - C$

C = Your Answer

X = The number you will receive

**Seat D Green Relay States 2026**

The perpendicular bisector of the segment with end points (3, 6) and (7, 4), is written in the form  $Ax + By = C$ . A, B, and C are relatively prime and  $A > 0$ . Find the value of  $A + B + C$ .

Pass back:  $D^3 - X - 7$

D = Your Answer

X = The number you will receive

**Seat E Green Relay States 2026**

If  $a \# b = aba^b - bab^a$ , find the value of  $(3 \# 2)$ .

Pass in:  $E^3 - X$

E = Your answer

X = The number you will receive

**Seat A Pink Relay States 2026**

Find the value of  $y^{-x}$  given  $0.5x - 0.2y = -1.9$  and  $0.2x + 0.3y = 0$

Pass back:  $\frac{32}{A}$                       A = Your answer

**Seat B Pink Relay States 2026**

The sum of the squares of two positive integers is 1753 and the difference of their squares is 295. Find the product of these two integers.

Pass back:  $\frac{B}{2X}$                       B = Your answer                      X = The number you will receive

**Seat C Pink Relay States 2026**

Find the surface area of a cube in square inches if its volume is 1331 cubic inches.

Pass back:  $X + C$                       C = Your answer                      X = The number you will receive

**Seat D Pink Relay States 2026**

How many 5-letter sequences can be formed using the letters of the word TRIANGLE, if each arrangement has 2 vowels and 3 consonants?

Pass back:  $\sqrt{D} + 2\sqrt{X + 66} + 6$                       D = Your answer                      X = The number you will receive

**Seat E Pink Relay States 2026**

If you use the eight digits 1, 3, 4, 5, 6, 7, 8, 9 each once and only once to form 4 two-digit prime numbers, what will be the largest sum the four prime numbers?

Pass in:  $\frac{E}{X-1}$                       E = Your answer                      X = The number you will receive

**Seat A Yellow Relay States 2026**

Find the value of  $x^{-y}$  given  $0.4x + 0.5y = 0.3$  and  $0.3x - 0.2y = 0.8$ .

Pass back:  $A^2$                       A = Your answer

**Seat B Yellow Relay States 2026**

The sum of the squares of two positive integers is 1649 and the difference of their squares is 399. Find the product of these two integers.

Pass back:  $\frac{B}{X}$                       B = Your answer                      X = The number you will receive

**Seat C Yellow Relay States 2026**

Find the surface area of a cube in square inches if its volume is 1728 cubic inches.

Pass back:  $C - 4X$                       C = Your answer                      X = The number you will receive

**Seat D Yellow Relay States 2026**

How many 5-letter sequences can be formed using the letters of the word MATRICES, if each arrangement has 2 vowels and 3 consonants?

Pass back:  $\frac{D}{2\sqrt{X}}$                       D = Your answer                      X = The number you will receive

**Seat E Yellow Relay States 2026**

If you use the eight digits 1, 2, 3, 4, 5, 6, 7, 9 each once and only once to form 4 two-digit prime numbers, what will be the largest sum the four prime numbers?

Pass in: X - E                      E = Your answer                      X = The number you will receive

## Solution Sheet – Individuals Round 1

1.  $\frac{AB}{DE} = \frac{\text{Perimeter } \triangle ABC}{\text{Perimeter } \triangle DEF} \rightarrow \frac{8}{12} = \frac{36}{P} \rightarrow P = 54.$  **Ans. 54**

2. The sum of the natural numbers from 1 to n is  $\frac{1}{2}n(n+1)$ , so  $\frac{1}{2}n(n+1) = \frac{1}{2}(2027)(2028)$  or  $(2027)(1014)$ . The unit's digit of this number is the unit's digit of  $7(4)$  which is 8. **Ans. 8**

3. Let  $A = 9778895$ . We then have  $(A-1)(A+1) + (A+1)(A+4) - A(A+3) - (A+3)(A-1) \rightarrow A^2 - 1 + A^2 + 5A + 4 - A^2 - 3A - A^2 - 2A + 3 = 6.$  **Ans. 6**

### Individuals Round 2

1.  $.15x = .2y \rightarrow 15x = 20y \rightarrow 3x = 4y \rightarrow \frac{3}{4}x = y \cdot \frac{3}{4} = 75\%$  **Ans. 75**

2.  $X + 2 \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} = -\frac{1}{3} \begin{bmatrix} -6 & 0 \\ 9 & -3 \end{bmatrix} \rightarrow X + \begin{bmatrix} 2 & 0 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix}.$  **Ans.  $\begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix}$**

3. If  $3i$  is a root, then so is  $-3i$ .  $(x - 3i)(x + 3i) = x^2 + 9$ .  $(x + 1)(x^2 + 9) = x^3 + x^2 + 9x + 9$ . The  $b + c + d = 1 + 9 + 9 = 19$ . **Ans. 19**

### Individuals Round 3

1.  $4.8 + 0.8x = .56x - 2.4 \rightarrow .24x = -7.2 \rightarrow x = -30.$  **Ans. -30**

2. Using synthetic division:

2	2	0	0	0	0	-36
	4	8	16	32	64	
	2	4	8	16	32	28

**Ans. 28**

3. Using one of the right triangles formed by the diagonals and the sides of the rhombus and the Pyth. Thm. :  $x^2 + (x + 1)^2 = (x + 9)^2 \rightarrow x^2 + x^2 + 2x + 1 = x^2 + 18x + 81 \rightarrow x^2 - 16x - 80 = 0$ ,  $(x - 20)(x + 4) = 0$ . Thus  $x = 20$ , since  $-4$  can't work. **Ans. 20**

### Individuals Round 4

1.  $P(\text{Green}): 1 - \left(\frac{1}{8} + \frac{1}{24} + \frac{1}{2} + \frac{1}{4}\right) = 1 - \left(\frac{3}{24} + \frac{1}{24} + \frac{12}{24} + \frac{6}{24}\right) = 1 - \frac{22}{24} = \frac{1}{12}.$  **Ans. 1/12**

2.  $(2 + k)x + (2 - k)y = 15 \rightarrow (2 - k)y = -(2 + k)x + 15$ . Slope:  $\frac{-(2+k)}{2-k} = \frac{3}{2} \rightarrow -4 - 2k = 6 - 3k$ .  $k = 10$ . **Ans. 10**

3.  $\sqrt{14 + \sqrt{27 - \sqrt{x - 1}}} = 4 \rightarrow 14 + \sqrt{27 - \sqrt{x - 1}} = 16 \rightarrow \sqrt{27 - \sqrt{x - 1}} = 2 \rightarrow 27 - \sqrt{x - 1} = 4 \rightarrow \sqrt{x - 1} = 23 \rightarrow x - 1 = 529$ , so  $x = 530$ . **Ans. 530**

### Individuals Round 5

1.  $\frac{1}{2}x + \frac{2}{3} = \frac{3}{4}x + \frac{4}{5} \rightarrow \frac{2}{3} - \frac{4}{5} = \frac{3}{4}x - \frac{1}{2}x \rightarrow -\frac{2}{15} = \frac{1}{4}x \rightarrow -\frac{8}{15} = x$ . **Ans. - 8/15**
2.  $|x^3 - 3| < 51 \rightarrow -51 < x^3 - 3 < 51 \rightarrow -48 < x^3 < 54$ . The cube root of -48 is between -3 and -4, while the cube of 54 is between 3 and 4. Thus the integral solutions are -3, -2, -1, 0, 1, 2, and 3 for a total of 7 integral solutions. **Ans. 7**
3.  $\cos(-\theta) = \sin(-\theta) \rightarrow \cos\theta = -\sin\theta \rightarrow -1 = \frac{\sin\theta}{\cos\theta}$ ,  $\tan\theta = -1$ .  $\theta = \frac{3}{4}\pi, \frac{7}{4}\pi$ . **Ans.  $\frac{3}{4}\pi, \frac{7}{4}\pi$**

### Individuals Round 6

1. The volume of a prism is base area times the height.  $26(15) = 390$ . **Ans. 390**
2. Subtracting the first equation from the third:  $3x - 5y + 8z - 2x + 6y - 7z = 23 - 16$ , we get  $x + y + z = 7$ . If one solves for each, you will get (4, 1, 2). **Ans. 7**
3. The semi-major axis is 10. The distance to a focus is 8. So the semi-minor axis is 6 from the Pyth. Thm. Thus the equation:  $\frac{(x-7)^2}{100} + \frac{(y-7)^2}{36} = 1$ . **Ans.  $\frac{(x-7)^2}{10^2} + \frac{(y-7)^2}{6^2} = 1$**

### Team Round 1

1.  $\log(a^2 - 15a) = 2 \rightarrow a^2 - 15a = 100 \rightarrow a^2 - 15a - 100 = 0$ ,  $(a-20)(a+5) = 0$ . Thus  $a = 20$  or  $-5$ . **Ans. 20 or -5**
2. There would be 7 integers larger than the median 42.  $42 + 7 = 49$ . **Ans. 49**
3.  $V = e^3 = 1728 = 12^3$ , so  $e = 12$ . The surface area = 6 times the area of each face. Thus  $SA = 6(12)(12) = 6(144) = 864$ . **Ans. 864**
4. The sides of the rt triangle are  $x$ ,  $x + 7$ , and  $x + 8$ . By Pyth. Thm.:  $x^2 + (x+7)^2 = (x+8)^2 \rightarrow x^2 + x^2 + 14x + 49 = x^2 + 16x + 64 \rightarrow x^2 - 2x - 15 = 0 \rightarrow (x-5)(x+3) = 0$ . Thus  $x = 5$  or  $-3$ .  $-3$  is an extraneous root. So this is a 5-12-13 rt triangle and  $\tan T = \frac{x}{x+7} = \frac{5}{12}$ . **Ans. 5/12**
5. The constant term of  $(4x^3 - \frac{2}{x^2})^5$  is  ${}_5C_2(4x^3)^2(\frac{-2}{x^2})^3 = 10(16x^6)(\frac{-8}{x^6}) = 160(-8)$  **Ans. -1280**
6. Rewrite each of the ratios  $\frac{1}{2} : \frac{2}{5} : \frac{8}{9}$  with the same denominator:  $\frac{45}{90} : \frac{36}{90} : \frac{80}{90}$ . Let  $x$  be the multiple needed so that  $45x + 36x + 80x = 483$ ,  $161x = 483$ , so  $x = 3$ . The number of clear and green bottles would be  $45x + 80x = 125x$ .  $125(3) = 375$ . **Ans. 375**
7.  $524,288 = 2 \cdot 4^{n-1} \rightarrow 262,144 = 4^{n-1} \rightarrow 4^9 = 4^{n-1}$ . So  $n = 10$ . **Ans. 10**
8. There are  $\binom{6}{2} = 15$  possible results. There is only one pair of trig functions, which when squared and added together will equal 1.  $\sin^2 x + \cos^2 x = 1$ . So  $1/15$ . **Ans. 1/15**

## Team Round 2

1.  $V = \frac{1}{3} \pi r^2 h \rightarrow \frac{3V}{\pi h} = r^2 \rightarrow \pm \sqrt{\frac{3V}{\pi h}} = r$ . Since the volume exists, then  $r = \frac{\sqrt{3V\pi h}}{\pi h}$       **Ans.  $\frac{\sqrt{3V\pi h}}{\pi h}$**

2. Since  $\angle EBF$  is bisected, then  $5x - 36 = 4x - 7y$ ,  $x = 36 - 7y$ . Since  $m\angle CBD = 90$ , then  $4x - 7y + 3x + 2y = 90$ , or  $7x - 5y = 90$ . Now  $7(36 - 7y) - 5y = 90 \rightarrow 252 - 54y = 90 \rightarrow 54y = 162$ ,  $y = 3$ .  $x = 36 - 7(3) = 15$ .  $x + y = 15 + 3 = 18$ .      **Ans. 18**

3.  $0 - 6x - 12x + 15 + 6x = -9 \rightarrow -12x = -24$ , so  $x = 2$ .      **Ans. 2**

4.  $-\frac{20}{3} \leq -\frac{2}{3} - 2y \leq \frac{10}{3} \rightarrow -20 \leq -2 - 6y \leq 10 \rightarrow -18 \leq -6y \leq 12 \rightarrow 3 \geq y \geq -2$ .

**Ans.  $-2 \leq y \leq 3$**

5.  $\cos(x + y) = \cos x \cos y - \sin x \sin y = \frac{4}{5} \cdot \frac{-12}{13} - \frac{3}{5} \cdot \frac{-5}{13} = \frac{-33}{65}$ .      **Ans.  $-\frac{33}{65}$**

6. This is a 7-24-25 triangle. The area of the triangle is 84, which can be found by finding the area of the three triangles with height of  $r$ , radius of circle.  $\frac{1}{2}r(7 + 24 + 25) = 84 \rightarrow 28r = 84$ , so  $r = 3$ . Area of circle is  $9\pi$ .      **Ans.  $9\pi$**

7.  $0.05(x + 3) - 0.25(x + 4) = 0.125(x - 5) - 0.625(x - 6) \rightarrow \frac{1}{20}(x+3) - \frac{1}{4}(x+4) = \frac{1}{8}(x-5) - \frac{5}{8}(x-6)$   
 $2(x+3) - 10(x+4) = 5(x-5) - 25(x-6) \rightarrow 2x + 6 - 10x - 40 = 5x - 25 - 25x + 150 \rightarrow 12x = 159$ ,  
 $x = 159/12 = 53/4 = 13\frac{1}{4}$ .      **Ans.  $53/4$  ( $13\frac{1}{4}$ ) (13.25)**

8. The multiple of 180 above 2026 is 2160.  $(n - 2)180 = 2160$ .  $n - 2 = 12$ ,  $n = 14$ .      **Ans. 14**

### Blue Relay    Seat A

$21x^2 + 22x - 24 = (7x + 12)(3x - 2)$ .  $a + b + c + d = 7 + 12 + 3 - 2 = 20$ . Pass back:  $A - 6 = 14$

**A = 20, Pass back: 14**

### Blue Relay    Seat B

Let  $y$  = youngest child,  $m$  = middle child,  $e$  = eldest child.  $y + m = e + 9$ ,  $m + e = y + 13$ .  
 $y + m - e = 9$  and  $m + e - y = 13$ , so  $2m = 22$ ,  $m = 11$ . Pass:  $B^{X-B} = 11^{14-11} = 121(11) = 1331$ .

**B = 11, Pass back: 1331**

### Blue Relay    Seat C

$m\widehat{MI} = 78^\circ$ , so  $m\angle IML = 39$ , and  $m\angle IML = 141$ . Pass:  $10C - X = 1410 - 1331 = 79$ .

**C = 141, Pass back: 79**

**Blue Relay Seat D**

Midpoint: (4, 7). Slope of given segment:  $\frac{6-8}{3-5} = 1$ , slope of perp. bis. = -1. Equation:  $y + x = 11$   
 $A + B + C = 1 + 1 + 11 = 13$ . Pass:  $X + D + 3 = 79 + 13 + 3 = 95$ . **D = 13, Pass back: 95**

**Blue Relay Seat E**

$3 \# 2 = 3 \cdot 2 \cdot 3^2 + 2 \cdot 3 \cdot 2^3 = 54 + 48 = 102$ . Pass:  $3X - 2E = 3(95) - 2(102) = 285 - 204 = 81$   
**E = 102, Pass in: 81**

**Green Relay Seat A**

$32x^2 - 52x + 15 = (8x - 3)(4x - 5)$ .  $F + g + h + j = 8 - 3 + 4 - 5 = 4$ . Pass:  $A^A = 4^4 = 256$   
**A = 4, Pass back 256**

**Green Relay Seat B**

Let  $y$  = youngest child,  $m$  = middle child,  $e$  = eldest child.  $y + m = e + 10$ ,  $m + e = y + 14$ .  
 $y + m - e = 10$  and  $m + e - y = 14$ , so  $2m = 24$ ,  $m = 12$ . Pass:  $\frac{3X}{B+4} + 5 = \frac{3(256)}{12+4} + 5 =$   
 $\frac{3 \cdot 16 \cdot 16}{16} + 5 = 48 + 5 = 53$ . **B = 12, Pass back: 53**

**Green Relay Seat C**

$m\widehat{MI} = 64^\circ$ , so  $m\angle IML = 32$ , and  $m\angle IML = 148$ . Pass:  $3X - C = 3(53) - 148 = 159 - 148 = 11$ .  
**C = 148, Pass back: 11**

**Green Relay Seat D**

Midpoint: (5, 5). Slope of given segment:  $\frac{6-4}{3-7} = -\frac{1}{2}$ , slope of perp. bis. = 2. Equation:  $y = 2x +$   
 $b$ ,  $5 = 2(5) + b$ ,  $b = -5$ ,  $2x - y = 5$ .  $A + B + C = 2 - 1 + 5 = 6$ . Pass:  $D^3 - X - 7 = 6^3 - 11 - 7 =$   
 $198$ . **D = 6, Pass back: 198**

**Green Relay Seat E**

Refer to Blue E.  $(3 \# 2) = 54 - 48 = 6$ . Pass:  $E^3 - X = 6^3 - 198 = 216 - 98 = 18$ .  
**E = 6, Pass in: 18**

**Pink Relay Seat A**

$0.5x - 0.2y = -1.9$  and  $0.2x + 0.3y = 0 \rightarrow 5x - 2y = -19$  and  $2x + 3y = 0$ ,  $y = -\frac{2}{3}x$ ,  $2y = -\frac{4}{3}x$ .  
Substituting:  $5x - (-\frac{4}{3}x) = -19 \rightarrow 15x + 4x = -57$ ,  $x = -3$ .  $2(-3) + 3y = 0$ ,  $y = 2$ .  $y^{-x} = 2^3 = 8$ .  
Pass:  $32/A = 32/8 = 4$ . **A = 8, Pass back: 4**

**Pink Relay Seat B**

$x^2 + y^2 = 1753$  and  $x^2 - y^2 = 295$ , adding:  $2x^2 = 2048$ , so  $x^2 = 1024$  and  $x = 32$ .  $1024 + y^2 = 1753$ , so  $y^2 = 729$  and  $y = 27$ .  $32(27) = 864$ . Pass:  $\frac{B}{2X} = \frac{864}{2(4)} = 108$ . **B = 864, Pass back: 108**

**Pink Relay Seat C**

$V = e^3 = 1331 = 11^3$ .  $6(11)^2 = 6(121) = 726$ . Pass:  $X + C = 108 + 726 = 834$ .

**C = 726, Pass back: 834**

**Pink Relay Seat D**

${}_3C_2 \cdot {}_5C_3 \cdot 5! = 3 \cdot 10 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 30(120) = 3600$ . Pass:  $\sqrt{D} + 2\sqrt{X + 66} + 6 = \sqrt{3600} + 2\sqrt{834 + 66} + 6 = 60 + 60 + 6 = 126$ .

**D = 3600, Pass back: 126**

**Pink Relay Seat E**

4,6,8 or 5 cannot be the unit's digit. So 9,7,3 or 1 have to be the ten's digit. 89, 67, 53, 41 could work, sum = 250. 89, 61, 53, 47 could also work, sum = 250. 83, 67, 59, 41 could work, sum = 250. 83, 61, 59, 47 could work, sum = 250. No matter which way you add these, the ten's digits add to 23 and the unit's digits add to 20. Pass:  $\frac{E}{X-1} = \frac{250}{126-1} = 2$ . **E = 250, Pass in: 2**

**Yellow Relay Seat A**

$0.4x + 0.5y = 0.3$  and  $0.3x - 0.2y = 0.8 \rightarrow 4x + 5y = 3$  and  $3x - 2y = 8 \rightarrow 8x + 10y = 6$  and  $15x - 10y = 40 \rightarrow$  Adding:  $23x = 46$ ,  $x = 2$ .  $4(2) + 5y = 3$ ,  $y = -1$ .  $x^{-y} = 2^1 = 2$ . Pass:  $A^2 = 4$ .

**A = 2, Pass back: 4**

**Yellow Relay Seat B**

$x^2 + y^2 = 1649$  and  $x^2 - y^2 = 399$ , adding:  $2x^2 = 2048$ , so  $x^2 = 1024$  and  $x = 32$ .  $1024 + y^2 = 1753$ , so  $y^2 = 625$  and  $y = 25$ .  $32(25) = 800$ . Pass:  $B/X = 800/4 = 200$ .

**B = 800, Pass back: 200**

**Yellow Relay Seat C**

$V = e^3 = 1728 = 12^3$ , so  $e = 12$ .  $6(144) = 864$ . Pass:  $C - 4X = 864 - 4(200) = 64$ .

**C = 864, Pass back: 64**

**Yellow Relay Seat D**

${}_3C_2 \cdot {}_5C_3 \cdot 5! = 3 \cdot 10 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 30(120) = 3600$ . Pass:  $\left(\frac{D}{2\sqrt{X}}\right) = \left(\frac{3600}{2\sqrt{64}}\right) = \frac{1800}{8} = 225$ .

**D = 3600, Pass back: 225**

## Yellow Relay Seat E

2, 4, 6 or 5 cannot be the unit's digit. 1, 3, 7, or 9 are. 67, 59, 41, 23 works, sum is 190. 67, 53, 41, 29 works, sum is 190. 61, 59, 47, 23 works, sum is 190. 61, 53, 47, 29 works, sum is 190. No matter which way you make these the ten's digits add to 17 and the unit's digits add to 20.

Pass:  $X - E = 225 - 190 = 35$ .

**E = 190, Pass in:35**

Answer Sheet States 2026

Individuals Round 1

1. 54

2. 8

3. 6

Individuals Round 4

1. 1/12

2. 10

3. 530

Team Round 1

1. 20, -5

4. 5/12

7. 10

2. 49

5. -1280

8. 1/15

3. 864

6. 375

Blue Relay

A = 20

B = 11

C = 141

D = 13

E = 102

Pass in

81

Pink Relay

A = 8

B = 864

C = 726

D = 3600

E = 250

Pass in

2

Individuals Round 2

1. 75 (75%)

2.  $\begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix}$

3. 19

Individuals Round 5

1. -8/15

2. 7

3.  $\frac{3}{4}\pi, \frac{7}{4}\pi$

Team Round 2

1.  $\frac{\sqrt{3V\pi h}}{\pi h}$

2. 18

3. 2

4.  $-2 \leq y \leq 3$

5. -33/65

6.  $9\pi$

7.  $53/4$  ( $13\frac{1}{4}$ ) (13.25)

8. 14

Green Relay

A = 4

B = 12

C = 148

D = 6

E = 6

Pass in

18

Yellow Relay

A = 2

B = 800

C = 864

D = 3600

E = 190

Pass in

35

Individuals Round 3

1. -30

2. 28

3. 20

Individuals Round 6

1. 390

2. 7

3.  $\frac{(x-7)^2}{10^2} + \frac{(y-7)^2}{6^2} = 1$