Individuals Round 1 States 2023

3 pts 1. Three pennies, showing 2 heads and 1 tail, lie on a table. Two coins are selected at random and turned over. What is the probability that all three coins are now all the same?

Ans. ____________________

4 pts 2. For all integers a, when ka is subtracted from $49a^2 + 16$, the result is a perfect square. Find the value(s) of k.

Ans. ____________________

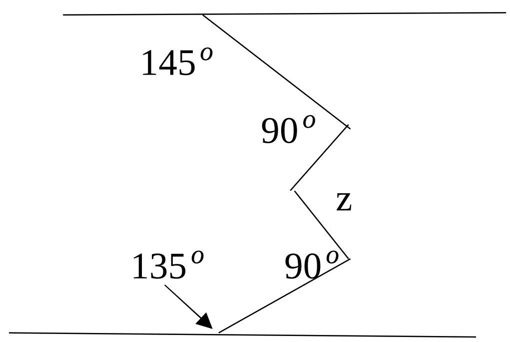
5 pts 3. Find all real value(s) of x such that:

$$\log_2(\log_2 x) = \log_4(\log_4 x)$$

Ans. ____________________

Individuals Round 2 States 2023

3 pts 1. Find the measure of angle z in the diagram, given that the two horizontal segments are parallel.



Ans. ____________________

4 pts 2. Let T_n be a triangle with vertices (2n, 1), (2n + 2, 1) and (-4n, -3). Find the sum of the areas of the set of triangles $\{T_1, T_2, T_3, \ldots, T_{100}\}$.

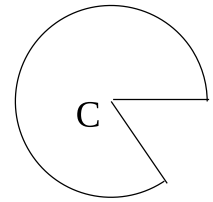
Ans. ____________________

5 pts 3. For the sequence 1, -2, 3, -4, 5, -6, 7, … , what is the difference between the sum of the first 400 terms and the sum of the first 200 terms.

Ans. ____________________

Individuals Round 3 States 2023

3 pts 1. An old video game featured a hungry creature in the form of a circle with a sector removed, its mouth. The missing piece has a radius of one and a central angle of 60^o. What is the exact perimeter of the creature? C is the center of the circle. Give answer in terms of π .



Ans. ____________________

4 pts 2. Allen has made 15 baskets out of 28 shots. Ben has made 20 out of 43. Allen takes c more shots and Ben takes d more shots. What is the least value of $c + d$, where $c \geq 1$ and $d \geq 1$, so that Allen and Ben could have the same average?

Ans. ______________________

5 pts 3. For how many integers n is $(n + i)^4$ an integer? $i = \sqrt{-1}$.

Ans. ________________________

Individuals Round 4 States 2023

3 pts 1. Let the operation # be defined by $a \# b = a^2 + 3b$. What is the value of

$$(2 \# 0) \# (0 \# 1)?$$

Ans. _______________________

4 pts 2. The function $f(x) = 0.5x + 10$ is evaluated at a positive number n. The result is then plugged into the same function and the result is n^2. Find all value(s) of n.

Ans. ____________________________

5 pts 3. Cube A has a surface area that is 125% more than the surface area of cube B. If the volume of B is x % less than the volume of A, find x to the nearest integer.

Ans. _______________________

Individuals Round 5 States 2023

3 pts 1. Determine the smallest positive integer value of *A* that will make (4410 times *A*) a perfect cube.

Ans. ____________________

4 pts 2. Use the recurrence formula $a_n = \frac{3a_{(n-1)} + 2a_{(n-2)}}{3}$ to find a_3, knowing that $a_0 = 2$ and $a_1 = 4$.

Ans. ____________________

5 pts 3. In the expansion of $(x + y + z)^7$, determine the coefficient of the term $x^3y^2z^2$.

Ans. ____________________

Individuals Round 6 States 2023

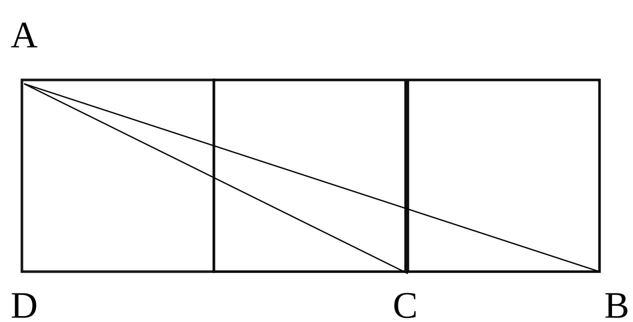
3 pts 1. $f(x) = \frac{1}{2}x^2$ and $g(x) = kx$. Find the value of k so that the amount of change in f(x) as x increases from -4 to -1 equals the amount of change in g(x) as x increases from -4 to -1.

Ans. ____________________

4 pts 2. What is the greatest possible remainder when a two-digit number is divided by the sum of its digits?

Ans. ________________

5 pts 3. Three squares are arranged as shown. Calculate $m\angle ABD + m\angle ACD$ in degrees.



Ans. ____________________

Team Round 1 States 2023

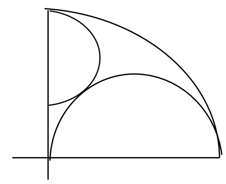
4 pts 1. The sides of a right triangle with integral lengths are in arithmetic progression. What is the length of the hypotenuse of the smallest triangle whose perimeter is greater than 2023?

(1) Ans. ______________ 4 pts

4 pts 2. Find the number of square units contained in the region bounded by the graphs of the functions $y = 3x$, $x = 3y$, and $3x + y = 30$.

(2) Ans. _____________ 4 pts

6 pts 3. The two semicircles in the figure are tangent to each other and their diameters are on the x-axis and y-axis. The arc has a radius of 2 and a center (0, 0). Find the length of the diameter of the smaller semicircle.



(3) Ans. _________________ 6 pts

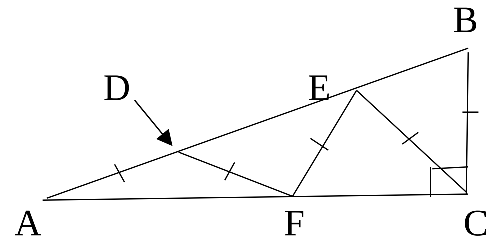
6 pts 4. Find the solution(s) when $x > 2/5$ for the equation $\log_x (5x - 2) = 3$.

(4) Ans. ___________________ 6 pts

6 pts 5. Bill Payer wrote six checks and addressed 6 envelopes for the corresponding six different bills. If he were to randomly put 1 check into each envelope, what is the probability that only two of the envelopes have the wrong check?

(5) Ans. ___________________ 6 pts

8 pts 6. In the diagram, ΔABC is a right triangle and $AD = DF = FE = EC = CB$. Find the measure of angle A.



(6) Ans. ___________________ 8 pts

8 pts 7. If $x + x^{-1} = 3$, $x^2 + x^{-2} = a$, and $x^3 + x^{-3} = b$, find the numerical value of $a + b$.

(7) Ans. __________________ 8 pts

8 pts 8. Let S denote the set of positive integers less than or equal to 18. How many subsets of S contain numbers whose sum is greater than 85?

(8) Ans. __________________ 8 pts

Team Round 2 States 2023

4 pts 1. Solve the systems below. State your answer in ordered pair form (x, y).

$$5\sqrt{x} - 2\sqrt{y} = 4\sqrt{2} \quad \text{and} \quad 2\sqrt{x} + 3\sqrt{y} = 13\sqrt{2}$$

(1) Ans. _______________ 4 pts

4 pts 2. The ratio of w to x is 4 : 3, of y to z is 3 : 2, and of z to x is 1 : 6.

What is the ratio of w to y?

(2) Ans. _______________ 4 pts

6 pts 3. Simplify completely, if $c \neq 4d$: $\frac{4}{c^2-4cd} - \frac{1}{cd-4d^2} - \frac{2}{cd}$.

(3) Ans. ___________________________ 6 pts

6 pts 4. A whispering room is built with an elliptical floor, such that when one stands at one of the foci, a whisper can be heard from the other focus. The dimensions of the room are 42 ft wide and 58 ft long. How far apart are the foci?

(4) Ans. _______________________ 6 pts

6 pts 5. Simplify: $\frac{\cos\theta\cot\theta}{1-\sin\theta} - 1$

(5) Ans. ______________________________ 6 pts

8 pts 6. Allen, Ben, and Chris run track. When all three raced, Allen beat Ben by 20 yds and Chris by 47 yds, and Ben finished 30 yds ahead of Chris. Find the distance that they raced.

(6) Ans. ____________________ 8 pts

8 pts 7. Given that $S_n = 1^7 + 2^7 + 3^7 + \cdots + n^7$ for all natural numbers n. Find the simplest form of the expression $\log_{\sqrt{7}}(S_{343} - S_{342})$.

(7) Ans. ______________________________ 8 pts

8 pts 8. Suppose that x is an acute angle for which sin x, sin 2x, and sin 4x form a strictly increasing arithmetic sequence. Compute the value of $\cos^3 x - \cos x$.

(8) Ans. ____________________ 8 pts

Seat A   Blue Relay   States 2023

Determine m given that $m > 0$ and the points $(m, 3)$ and $(1, m)$ lie on a line with slope m.

Pass back: $A^2 + 1$ A = Your answer

Seat B   Blue Relay   States 2023

A full quart bottle of liquid is 10% alcohol. A full 3-quart bottle is 30% alcohol. If the contents are mixed together, what is the percent of alcohol in the 4-quart mixture?

Pass back: X + B/5   B = Your answer   X = The number you will receive.

Seat C   Blue Relay   States 2023

An 8-15-17 triangle is inscribed in a circle. The circumference of the circle is $C\pi$. Find C.

Pass back: X + C   C = Your answer   X = The number you will receive.

Seat D   Blue Relay   States 2023

Find all real values of x such that: $2x + \sqrt{x} - 1 = 0$.

Pass back: $X\sqrt{D}$ D = Your answer   X = The number you will receive.

Seat E   Blue Relay   States 2023

Define a sequence of complex numbers by $Z_1 = 0$, $Z_{n+1} = (Z_n)^2 + i$ for $n \geq 1$. In the complex plane, how far from the origin is Z_{111}? $i = \sqrt{-1}$.

Pass in: $X - E^2$ E = Your answer   X = The number you will receive.

Seat A   Green Relay   States 2023

Determine m given that $m > 0$ and the points $(m, 5)$ and $(1, m)$ lie on a line with slope m.

Pass back: $A^2 + 1$ A = Your answer

Seat B   Green Relay   States 2023

A full 2-quart bottle of liquid is 20% alcohol. A full 6-quart bottle is 60% alcohol. If the contents are mixed together, what is the percent of alcohol in the 8-quart mixture?

Pass back: X + B/5   B = Your answer   X = The number you will receive.

Seat C   Green Relay   States 2023

A 9-12-15 triangle is inscribed in a circle. The circumference of the circle is $C\pi$. Find C.

Pass back: X + C   C = Your answer   X = The number you will receive.

Seat D   Green Relay   States 2023

Find all real values of x such that: $3x + 2\sqrt{x} - 1 = 0$.

Pass back: $(X - 1)\sqrt{D}$ D = Your answer   X = The number you will receive.

Seat E   Green Relay   States 2023

Define a sequence of complex numbers by $Z_1 = 0$, $Z_{n+1} = (Z_n)^2 + i$ for $n \geq 1$. In the complex plane, how far from the origin is Z_{119}? $i = \sqrt{-1}$.

Pass in: $X - E^2$ E = Your answer   X = The number you will receive.

Seat A   Pink Relay   States 2023

Find the sum of the values of y that satisfy $|y - 1.5|$ = y/2.

Pass back: 2A   A = Your answer

Seat B   Pink Relay   States 2023

A cell phone originally priced at $500 is discounted in succession 10%, 20% and 30%. Each discount was on the price at that time. What is the price of the phone after the third discount?

Pass back: B ÷ (X ÷ 2)   B = Your answer   X = The number you will receive

Seat C   Pink Relay   States 2023

How many diagonals are there in a regular 19-gon?

Pass back: C – 2X   C = Your answer   X = The number you will receive

Seat D   Pink Relay   States 2023

If 3 + 2i is one root of the quadratic function f(x) = x^2 + Ax + B, where A and B are real numbers, find A + B. i = $\sqrt{-1}$.

Pass back: D – X/2   D = Your answer   X = The number you will receive

Seat E   Pink Relay   States 2023

Find the sum of the infinite series: 5 + 10/3 + 20/9 + . . .

Pass Back: X + E   E = Your answer   X = The number you will receive

Seat A   Yellow Relay   States 2023

Find the sum of the values of y that satisfy $|y-4.5|$ = y/2.

Pass back: A/2   A = Your answer

Seat B   Yellow Relay   States 2023

A cell phone originally priced at $250 is discounted in succession10%, 20% and 30%. Each discount was on the price at that time. What is the price of the phone after the third discount?

Pass back: B ÷ (X ÷ 2)   B = Your answer   X = The number you will receive

Seat C   Yellow Relay   States 2023

How many diagonals are there in a regular 20-gon?

Pass back: C – 2X   C = Your answer   X = The number you will receive

Seat D   Yellow Relay   States 2023

If 2 + 3i is one root of the quadratic function f(x) = x^2 + Ax + B, where A and B are real numbers, find A + B. $i = \sqrt{-1}$.

Pass back: D – X/2   D = Your answer   X = The number you will receive

Seat E   Yellow Relay   States 2023

Find the sum of the infinite series: 15 + 10 + 20/3 + . . .

Pass Back: X + E   E = Your answer   X = The number you will receive

Solutions – Individuals Round 1 States 2023

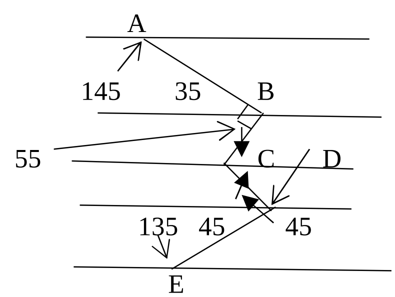
1. Consider that you have H_1, T, H_2. If H_1 is not turned but the other two are, you end up with 2 heads and a tail. The same result happens from H_2. If T is not turned and the others are, you end up with 3 heads. Answer 1/3. **Ans. 1/3**

2. $49a^2$ - ka + 16 is a perfect square. Knowing that $(7a \pm 4)^2$ creates two perfect squares, $49a^2 \pm 56a + 16$, k must equal $\pm$ 56. **Ans. $\pm$ 56**

3. $\log_2(\log_2 x) = \log_4(\log_2 x)^2 = \log_4(\log_4 x)$. Thus, $(\log_2 x)^2 = (\log_4 x) = \log_2 \sqrt{x}$ ➔ $(\log_2 x)^2 = \frac{1}{2}\log_2 x$ ➔ $(\log_2 x)^2 - \frac{1}{2}\log_2 x = 0$. $\log_2 x(\log_2 x - \frac{1}{2}) = 0$. Either (1) $\log_2 x = 0$ or (2) $\log_2 x = ½$. In (1) $2^o = x$, thus x = 1, since $\log_2 1 = 0$, then in the original equation $\log_2(\log_2 1) = \log_2 0$. Since we can't take log 0, x cannot = 1; or (2) $x = 2^{\frac{1}{2}} = \sqrt{2}$. **Ans. $\sqrt{2}$**

Individuals – Round 2

1. If we draw 3 more parallel lines as shown through B, C, and D, The 2 angles at B are 35 and 55. The upper angle at C is 55. The 2 angles at D are 45 and 45. So the lower angle at C is 45. The angles that make up Z add to 100.



Ans. 100

2. When n = 1, vertices are (2, 1), (4, 1) and (-4, -3), the area = 4. When n = 2, vertices are (4, 1), (6, 1) and (-8, -3) the area is 4. 100(4) = 400. **Ans. 400**

3. Adding each pair of terms gives -1, -1, -1, ... The sum of the first 400 terms is – 200 and the sum of the first 200 terms is -100. The difference is -200 – (-100) = **Ans. – 100**

Individuals – Round 3

1. With radius of 1, the circumference of the part of the circle shown is $\frac{5}{6}(2\pi(1) = \frac{5}{3}\pi$. The rest of the creature, its mouth is 2. **Ans. $\frac{5}{3}\pi$ + 2**

2. The denominators should be able to reduce for equal fractions. Allen: $\frac{15}{28}$ ➔ $\frac{15 or 16 or 17}{30}$ or $\frac{15 or 16 or 17 or 18 or 19 or 20}{33}$. Barry: $\frac{20}{43}$ ➔ $\frac{20 or 21}{44}$ or $\frac{20 or 21 or 22}{45}$ or $\frac{20 or 21 or 22 or 23 or 24 or 25}{48}$. Using these for minimal number of shots taken, Allen is $\frac{15}{33} = \frac{5}{11}$. Barry is $\frac{20}{44} = \frac{5}{11}$. Together they took 6 shots and missed all 6. **Ans. 6**

3. $(n+i)^4 = n^4 + 4n^3i + 6n^2i^2 + 4ni^3 + i^4$. $n^4 + 6n^2i^2 + i^4$ will produce an integer. $4n^3i + 4ni^3$ will not for $n \geq 1$. For what values of n would the expression equal 0?

$4n^3i + 4ni^3 = 0 \Rightarrow 4n^3i - 4ni = 0 \Rightarrow 4ni(n^2 - 1) = 0$. n = 0, 1, -1. **Ans. 3**

Individuals – Round 4

1. $2 \# 0 \Rightarrow 2^2 + 3(0) = 4$. $0 \# 1 \Rightarrow 0^2 + 3(1) = 3$. $4 \# 3 \Rightarrow 4^2 + 3(3) = 25$. **Ans. 25**

2. $f(x) = \frac{1}{2}x + 10$. $f(n) = \frac{1}{2}n + 10$. $f(\frac{1}{2}n + 10) = \frac{1}{2}(\frac{1}{2}n + 10) + 10 = \frac{1}{4}n + 15 = n^2 \Rightarrow$

$4n^2 - n - 60 = 0 \Rightarrow (n-4)(4n+15) = 0$. n = 4, since it has to be positive. **Ans. 4**

3. Surface area of cube A = $6a^2$, for B = $6b^2$, where a and b are sides of each. $6a^2 = \frac{9}{4}(6b^2)$.

Thus $a = \frac{3}{2}b$. $\frac{\textit{volume of } B}{\textit{volume of } A} = \frac{b^3}{\frac{27}{8}b^3} = \frac{8}{27} = .296$. $(100 - 29.6)\% = 70.4\%$. **Ans. 70%**

Individuals – Round 5

1. 4410 = 21(21)(10). To make this a cube: (21)(10)(10) = 2100. **Ans. 2100**

2. $a_2 = \frac{3\cdot4+2\cdot2}{3} = 16/3$. $a_3 = \frac{3\left(\frac{16}{3}\right)+2\cdot4}{3} = \frac{24}{3} = 8$. **Ans. 8**

3. $(x+y+z)^7 = ((x+y)+z)^7 \Rightarrow {}_7C_2(x+y)^5z^2 = {}_7C_2\cdot{}_5C_2\cdot x^3y^2z^2$. The coefficients: $\frac{7\cdot6}{2}\cdot\frac{5\cdot4}{2} = 7\cdot6\cdot5 = 210$. **Ans. 210**

Individuals – Round 6

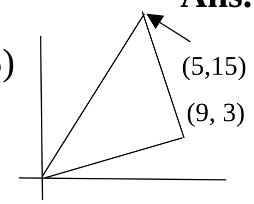
1. $f(x) = \frac{1}{2}x^2$. $f(-4) = \frac{1}{2}(-4)^2 = 8$. $f(-1) = \frac{1}{2}(-1)^2 = \frac{1}{2}$. $8 - \frac{1}{2} = 7\frac{1}{2}$. $g(x) = kx$. $g(-4) = -4k$. $g(-1) = -k$. $-4k - (-k) = 7\frac{1}{2} \Rightarrow -3k = 7\frac{1}{2}$, $k = -2\frac{1}{2}$. **Ans. $-2\frac{1}{2}$**

2. Through guess and check: larger numbers would have larger remainders, numbers with corresponding remainders: $\frac{99}{18}$, 9; $\frac{98}{17}$, 13; $\frac{89}{17}$, 4; $\frac{88}{16}$, 8; $\frac{97}{16}$, 1; $\frac{79}{16}$, 15. The rest of the sums are 15 or less. So 15 is the largest remainder. **Ans. 15**

3. tan $\angle$ABD = 1/3, tan $\angle$ACD = 1/2. tan $(\alpha+\beta)$ = $\dfrac{\tan\alpha\tan\beta}{1-\tan\alpha\tan\beta}$ ➔ $\dfrac{(1/3)+(1/2)}{1-(1/3)(1/2)}$ = $\dfrac{5/6}{5/6}$ = 1.
Thus $\alpha+\beta$ = 45^o. **Ans. 45^o**

Team Round 1 States 2023

1. The right triangle with arithmetic progression sides is the 3-4-5. 2020/12 = 168+ , so the triangle is a 169(3-4-5) , the hypotenuse is 5(169) = 845. **Ans. 845**



2. Using the graphs in pairs, you will find the ordered pairs (0, 0), (5, 15) and (9, 3). Graphed, it looks like a rt.Δ. The slope from (0, 0) to (9, 3) is 1/3, from (9, 3) to (5, 15) is -3. So we do have a rt.Δ. The length of the first leg is $3\sqrt{10}$, the other is $4\sqrt{10}$. The area is $\frac{1}{2}(3\sqrt{10})(4\sqrt{10})$ = 60. Other methods such as using Hero's formula will work as well. The quickest method is using determinants:

$\frac{1}{2}\left\|\begin{matrix}0 & 0 & 1\\ 9 & 3 & 1\\ 5 & 15 & 1\end{matrix}\right\| = \frac{1}{2}(135 - 15) = \frac{1}{2}(120) = 60.$ **Ans. 60**

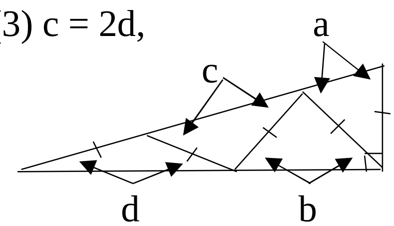
3. If you connect from the center of the x-axis semicircle through the point of tangency of the two semicircles to the y-axis, you will be at the center of the small semicircle. In this right triangle, if you let the radius of the small semicircle be r, then you get $(2 - r)^2 + 1^2 = (1 + r)^2$. $4 - 4r + r^2 + 1 = 1 + 2r + r^2$ ➔ 4 = 6r, r = 2/3. Diameter of small semicircle = 4/3. **Ans. 4/3**

4. $\log_x(5x - 2) = 3$ ➔ $x^3 - 5x + 2 = 0$ ➔ $(x - 2)(x^2 + 2x - 1) = 0$. x = 2, or x = $\dfrac{-2\pm\sqrt{4+4}}{2}$ = $-1\pm\sqrt{2}$. Because x > 2/5, then $-1+\sqrt{2}$ can work, since $\sqrt{2}$ = 1.414. **Ans. 2, -1 + $\sqrt{2}$**

5. The prob. of the first envelope to get the right check is 1/6, for the second is 1/5, for the third is 1/4 and for the fourth 1/3. For the fifth to get the wrong check is 1/2 and the last envelope is 1. The number of ways the combination of checks can be switched is $\binom{6}{4}$. Thus:

$\frac{1}{6}\cdot\frac{1}{5}\cdot\frac{1}{4}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot 1\cdot\binom{6}{4} = \frac{3\cdot 5}{6\cdot 5\cdot 4\cdot 3\cdot 2} = \frac{1}{48}$ **Ans. 1/48**

6. Using the figure labeled at right: (1) a + d = 90, (2) a + c = 2b, (3) c = 2d, b + 180 – 2a = 90, so (4) 2a –b = 90. Now: In (2) using (3) and (4): a + 2d = 2(2a – 90) ➔ a + 2d = 4a – 180 ➔ 2d = 3a – 180. Using (1); 2(90 – a) = 3a – 180 ➔ 180 – 2a = 3a – 180 ➔ 360 = 5a, so a = 72. Thus, d = 18.



Ans. 18

7. $x + x^{-1} = 3$, $(x + x^{-1})^2 = 9 \Rightarrow x^2 + 2 + x^{-2} = 9$, so $x^2 + x^{-2} = 7 = a$. $(x + x^{-1})^3 = 27 \Rightarrow$
$x^3 + 3x + 3x^{-1} + x^{-3} = 27 \Rightarrow x^3 + 3(x + x^{-1}) + x^{-3} = 27 \Rightarrow x^3 + 3(3) + x^{-3} = 27 \Rightarrow$
$x^3 + x^{-3} = 18 = b$. $a + b = 7 + 18 = 25$. **Ans. 25**

8. The sum of all the numbers from 1 to 18 is $(1 + 18)\frac{18}{2} = 19(9) = 171$. Half of 171 is $85\frac{1}{2}$. For every subset with a sum of 85, it will have a complement less than 85. For every subset with a sum which is above 85, it will have a complement with a sum below 85. There are 2^{18} subsets, so there will be $2^{18}/2 = 2^{17}$ subsets greater than 85. Testing out this logic using the first four natural numbers. For a set whose only element is 1, the sum is 1. Half the sum is 1/2. The # of subsets is $2^1 = 2$. The number of subsets with sum greater than 1/2 is 1, Which is 1/2 of the subsets. Using 1 and 2: the sum is 3 and half that sum is 1½. The subsets which have a sum greater than 1½ are {1,2} and {2}, which is 2. There are $2^2 = 4$ subsets in all. 2/4 = 1/2. Using 1, 2, 3: 6/2 = 3. Subsets whose sum ≥ 3; {1,2}, {1,3}, {2,3}, {3}. There are 4. There are $2^3 = 8$ subsets in all. 4/8 = 1/2. You can try 5 and 6, but you will find that if half the sum of the numbers in the set is a whole number, the number of subsets whose sum is greater than or equal to that is half the subsets. If half the sum is a whole number plus 1/2, the number of subsets greater than or equal to the whole number will be half the sum of the number of subsets of the set. $2^{17} = 2^{10} \cdot 2^7 = 1024(128) = 131{,}072$ **Ans. 131,072**

Team 2

1. $5\sqrt{x} - 2\sqrt{y} = 4\sqrt{2} \Rightarrow 15\sqrt{x} - 6\sqrt{y} = 12\sqrt{2}$. $19\sqrt{x} = 38\sqrt{2}$, $\sqrt{x} = 2\sqrt{2}$, x = 8.

$2\sqrt{x} + 3\sqrt{y} = 13\sqrt{2} \Rightarrow 4\sqrt{x} + 6\sqrt{y} = 26\sqrt{2}$. $2(2\sqrt{2}) + 3\sqrt{y} = 13\sqrt{2}$, $\sqrt{y} = 3\sqrt{2}$, y = 18.

Ans. (8. 18)

2. $\frac{w}{x} = \frac{4}{3}$, (1) 3w = 4x. $\frac{y}{z} = \frac{3}{2}$, (2) 2y = 3z. $\frac{z}{x} = \frac{1}{6}$, (3) 6z = x. (3) ==> (1): 3w = 4(6z) , w = 8z

In (2): $z = \frac{2}{3}y$. Thus $w = 8(\frac{2}{3}y) = \frac{16}{3}y \Rightarrow \frac{w}{y} = \frac{16}{3}$. **Ans. 16:3**

3. $\frac{4}{c^2 - 4cd} - \frac{1}{cd - 4d^2} - \frac{2}{cd} = \frac{4}{c(c-4d)} - \frac{1}{d(c-4d)} - \frac{2}{cd} = \frac{4d}{dc(c-4d)} - \frac{1c}{cd(c-4d)} -$
$\frac{2(c-4d)}{cd(c-4d)} = \frac{4d - c - 2c + 8d}{cd(c-4d} = \frac{-3c+12d}{cd(c-4d)} = \frac{-3}{cd}$. **Ans.** $\frac{-3}{cd}$

4. The vertex is 29 units from the center. The minor axis is 21. This will make a 20-21-29 right triangle, with the focus 20 units from center. Thus 40 units between the foci. **Ans. 40 ft**

5. $\dfrac{\cos\theta\cot\theta}{1-\sin\theta}$ - 1 = $\dfrac{\cos\theta\dfrac{\cos\theta}{\sin\theta}}{1-\sin\theta}$ - 1 = $\dfrac{\cos^2\theta}{\sin\theta(1-\sin\theta)}$ - 1 = $\dfrac{1-\sin^2\theta}{\sin\theta(1-\sin\theta)}$ - 1 =
$\dfrac{(1-\sin\theta)(1+\sin\theta)}{\sin\theta(1-\sin\theta)}$ - 1 = $\dfrac{1+\sin\theta}{\sin\theta}$ - 1 = cscθ + 1 – 1 = csc θ. **Ans. csc θ**

6. Use rt = d. Let Allen's rate be a, Ben's rate be b, and Chris's rate be c. After Allen finished: at = d, bt = d – 20 and ct = d – 47. When Ben finished, bT = d and cT = d – 30.

Setting these into a ratio: (1) $\dfrac{at}{bt}=\dfrac{d}{d-20}$, (2)$\dfrac{bt}{ct}=\dfrac{d-20}{d-47}$, (3)$\dfrac{at}{ct}=\dfrac{d}{d-47}$, (4)$\dfrac{bT}{cT}=\dfrac{d}{d-30}$.

Using(2) and (4): $\dfrac{d-20}{d-47}=\dfrac{d}{d-30}$ ➔ d^2 - 47d = d^2 - 50d + 600 = 0. 3d = 600. **Ans. 200 yds**

7. S_{343} - S_{342} = 343^7, therefore $\log_{\sqrt{7}}$ (S_{343} - S_{342}) = $\log_{\sqrt{7}} 343^7$ = 7 $\log_7 343^2$ =
14 $\log_7$ 343 = 14(3) = 42. **Ans. 42**

8. sin 4x – sin 2x = sin 2x – sin x ➔ 2 sin 2x cos 2x – sin 2x = sin 2x – sin x ➔2 sin 2x cos 2x – 2 sin 2x = - sin x ➔ 2 sin 2x(cos 2x – 1) = - sin x ➔
4 sin x cos x(2 $\cos^2$x – 2) = - sin x ➔ 8 sin x $\cos^3$x – 8 sin x cos x = - sin x ➔
8 $\cos^3$x – 8cos x = -1 ➔ $\cos^3$x – cos x = - 1/8. **Ans. – 1/8**

Blue Relay – Seat A

m = $\dfrac{3-m}{m-2}$ ➔ m^2 - m = 3 – m ➔ m^2 - 3 = 0. m = $\pm\sqrt{3}$. Pass: ($\pm\sqrt{3}$) + 1 = 4.

A = $\sqrt{3}$, Pass. 4

Blue Relay – Seat B

1(.10) + 3(.30) = 4x ➔ 1 = 4x, x = 1/4 = 25%. Pass: 4 + (25)/5 = 9. **B = 25, Pass: 9**

Blue Relay – Seat C

The triangle is a right triangle, so its hypotenuse is the diameter. C = 17π. C = 17.

Pass: 17 + 9 = 26. **C = 17, Pass: 26**

Blue Relay – Seat D

2x + $\sqrt{x}$ - 1 = 0 ➔ $\sqrt{x}$ = 2x – 1 ➔ x = $4x^2$ - 4x + 1 ➔ $4x^2$ - 5x + 1 = 0 ➔ (4x – 1)(x – 1) = 0.

x = 1 or 1/4. 1 does not work. D = 1/4. Pass: 26 + $\sqrt{1}$ = 13. **D = ¼, Pass: 13**

Blue Relay – Seat E

$Z_1 = 0$. $Z_2 = 0^2 + i = i$, $Z_3 = i^2 + i = -1 + i$, $Z_4 = (-1 + i)^2 + i = 1 - 2i - 1 + i = -i$,

$Z_5 = (-i)^2 + i = -1 + i$. Every odd power yields $-1 + i$. $\sqrt{2}$ from origin. E = $\sqrt{2}$.

Pass: $13 - (\sqrt{2})^2 = 11$. **E = $\sqrt{2}$, Pass: 11**

Green Relay – Seat A

$m = \frac{5-m}{m-1}$ ➔ $m^2 - m = 5 - m$ ➔ $m^2 = 5$, $m = \sqrt{5}$. Pass: $(\sqrt{5})^2 = 1 = 6$. **A = $\sqrt{5}$, Pass: 6**

Green Relay – Seat B

2(.20) + 6(.60) = 8x ➔ 4 = 8x, x = 50%. B = 50, Pass: 6 + 50/5 = 16. **B = 50, Pass: 16**

Green Relay – Seat C

Refer to Blue Seat C: C = 15π. C = 15, Pass: 16 + 15 = 31. **C = 15, Pass: 31**

Green Relay – Seat D

$3x + 2\sqrt{x} - 1 = 0$ ➔ $2\sqrt{x} = 1 - 3x$ ➔ $4x = 1 - 6x + 9x^2$ ➔ $9x^2 - 10x + 1 = 0$ ➔

(9x – 1(x – 1) = 0. x = 1 or 1/9. x $\neq$ 1. D = 1/9. Pass: $(31 - 1)(\sqrt{\frac{1}{9}}) = 10$. **D = 1/9, Pass: 10**

Green Relay – Seat E

Same as Blue Seat E: A = $\sqrt{2}$. Pass: $10 - (\sqrt{2})^2 = 8$. **E = $\sqrt{2}$, Pass: 8**

Pink Relay – Seat A

$|y - 1.5| = y/2$ ➔ $y - \frac{3}{2} = \pm\frac{1}{2}y$. Either (1) $\frac{1}{2}y = \frac{3}{2}$, so y = 3, or $\frac{3}{2}y = \frac{3}{2}$, so y = 1. 3 + 1 = 4.

A = 4. Pass 2(4) = 8 **A = 4, Pass: 8**

Pink Relay – Seat B

500(.9)(.8)(.7) = 252 = B, Pass: $252 \div (8 \div 2) = 63$. **B = 252, Pass: 63**

Pink Relay – Seat C

$\frac{19(16)}{2} = 19(8) = 152$ = C. Pass: 152 – 2(63) = 26. C **= 152, Pass: 26**

Pink Relay – Seat D

If 3 - 2i is one root and both roots are real, then 3 + 2i is the other root. Their sum is 6 and their product is 13. So the equation has form $x^2 - 6x + 13 = 0$. Sum is 7 = D. Pass: 7 – 26/2 = - 6.

D = 7, Pass: -6

Pink Relay – Seat E

Sum = $\frac{5}{1-2/3}$ **=** $\frac{5}{1/3}$ **=** 15 = E. Pass: -6 + 15 = 9. **E = 15, Pass: 9**

Yellow Relay – Seat A

$|y-4.5|$ = y/2 ➔ y - $\frac{9}{2}$ = ± $\frac{1}{2}$y. Either $\frac{1}{2}$y = $\frac{9}{2}$, so y = 9 or $\frac{3}{2}$y = $\frac{9}{2}$, so y = 3. 3 + 9 = 12 = A.

Pass: $12 \div 2$ = 6. **A = 12, Pass: 6**

Yellow Relay – Seat B

250(.9)(.8)(.7) = 126 = B. Pass: 126 ÷ (6 ÷ 2) = 42. **B = 126, Pass: 42**

Yellow Relay – Seat C

$\frac{20(17)}{2}$ = 170 = A. Pass: 170 – 2(42) = 86. **C = 170, Pass: 86**

Yellow Relay – Seat D

2 ± 3*i*. Sum is 4. Product is 13. Equation: x^2 - 4x + 13 = 0. D = 9. Pass: 9 – 86/2 = -34.

D = 9, Pass: - 34

Yellow Relay – Seat E

$\frac{15}{1-2/3}$ **=** $\frac{15}{1/3}$ **=** 45 = E. Pass: -34 + 45 = 11. **E = 45, Pass: 11**

Answer Sheet States 2023

Individuals Round 1

1. **1/3 or $0.\overline{3}$ or $33\frac{1}{3}\%$ or $33.\overline{3}\%$**
2. **$\pm$ 56**
3. **$\sqrt{2}$ or $2^{1/2}$**

Individuals Round 2

1. **100 or 100°**
2. **400 or 400 $units^2$**
3. **-100 or 100**

Individuals Round 3

1. $\frac{5}{3}\pi + 2$ or $(5\pi + 6)/3$ or $\left(1\frac{2}{3}\right)\pi + 2$
2. **5**
3. **3**

Individuals Round 4

1. **25**
2. **4**
3. **70** or **70%**

Individuals Round 5

1. **2100**
2. **8**
3. **210**

Individuals Round 6

1. **– 2.5** or $-\frac{5}{2}$ or $-2\frac{1}{2}$ or 2.5 or $\frac{5}{2}$ or $2\frac{1}{2}$ or ± 2.5 or $\pm\frac{5}{2}$ or $\pm 2\frac{1}{2}$
2. **15**
3. **45^o** or **45**

Team Round 1

1. **845**
2. **60**
3. **4/3**
4. **2, -1+$\sqrt{2}$**
5. **1/48**
6. **18** or **18^o**
7. **25**
8. **131,072**

Team Round 2

1. **(8, 18)**
2. **16:3** or **16/3** or **16 to 3**
3. **– 3/cd**
4. **40** or **40ft**
5. **cscθ** or **1/sinθ**
6. **200** or **200 yds**
7. **42**
8. **– 1/8**

Blue Relay

A = $\sqrt{3}$, Pass 4

B = 25, Pass 9 or 25%

C = 17, Pass 26

D = 1/4, Pass 13

Green Relay

A = $\sqrt{5}$, Pass 6

B = 50, Pass 16 or 50%

C = 15, Pass 31

D = 1/9, Pass 10

Pink Relay

A = 4, Pass 8

B = 252, Pass 63

C = 152, Pass 26

D = 7, Pass – 6

Yellow Relay

A = 12, Pass 6

B = 126, Pass 42

C = 170, Pass 86

D = 9, Pass – 34

E = $\sqrt{2}$**, Pass 11** **E =** $\sqrt{2}$**, Pass 8** **E = 15, Pass 9** **E = 45, Pass 11**